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RONALD STEPHEN SLOBODA

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled RADIATIVE MUON CAPTURE IN CALCIUM submitted by RONALD STEPHEN SLOBODA in partial fulfilment of the requirements for the degree of DOCTOR OF SCIENCE in THEORETICAL PHYSICS.

DEDICATION

To Candy

ABSTRACT

Motivated by the observation that the photon asymmetry in radiative muon capture is determined by terms of $O(1/m^2)$ in an expansion in powers of the nucleon mass m, a new calculation of radiative muon capture in ^{40}Ca was made which is consistent, within the standard theory, to $O(1/m^2)$. The Hamiltonian was expanded via a Foldy-Wouthuysen reduction through $O(1/m^2)$ instead of the usual $O(1/m)$ to allow for the first time a calculation of the $O(1)xO(1/m^2)$ contributions to the rate. The results show that the $O(1/m^2)$ terms are definitely necessary but that the most significant ones arise from the $O(1/m)xO(1/m)$ contributions which can be obtained from the usual $O(1/m)$ Hamiltonian.

A number of nuclear effects were also considered including an improved giant dipole resonance model and a model using Hartree-Fock wave functions including spin-orbit coupling. As in most previous calculations the ratio of radiative to ordinary rates and the photon asymmetry are relatively insensitive to the details of the model, though the absolute rates can be quite sensitive. Fits to the available data to extract the induced pseudoscalar coupling g_P were also done.

In an improvement to the closure approximation, the photon spectrum was expanded about the average maximum photon energy k_m and the correction terms evaluated using for one a modified Thomas-Reiche-Kuhn sum rule. The resulting rate is much less dependent on k_m than the usual

closure result. The ratio k_m/ν appropriate for closure calculations, with ν the average neutrino energy, was determined and found to be approximately constant and, with correction terms included, somewhat higher than values previously used. By similar techniques a consistency relation was derived which can be solved to explicitly estimate "physical" values of k_m and ν.

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I.

INTRODUCTION

Radiative muon capture (RMC) is a relatively rare process first observed in iron by Conforto et al (Co 62). The measured branching ratio for radiative capture of roughly 10^{-4} times the ordinary capture rate was later verified by Chu et al (Ch 65), using a copper target. Subsequent experimental effort largely focussed on the ^{40}Ca nucleus. The ^{40}Ca photon spectrum was initially measured by Conversi et al (Co 64) at CERN; the photon spectrum plus asymmetry have since been obtained by Di Lella, Rosenstein et al (Di 71, Ro 73) at Nevis, and Hart et al (Ha 77) at SREL. An experiment proposed for TRIUMF (Ha 75) will look at the photon spectrum and asymmetry for ^{40}Ca and ^{208}Pb.

Early theoretical studies (Hu 57, Ma 59, Op 64) were confined to radiative capture on a free proton since no complicating nuclear structure effects appear. Experiments using ^{1}H targets are barely feasible because of the extremely low absolute capture rates, so theoretical efforts naturally shifted to complex nuclei. One of the first theoretical studies of radiative capture on nuclei was carried out by Primakoff (Pr 59), who used the closure approximation and considered only the muon radiating diagram. Rood and Tolhoek (Ro 65) made a detailed investigation of RMC in ^{16}O and ^{40}Ca using the impulse approximation. References to the "standard" theory of radiative muon capture made here really refer to the Rood

and Tolhoek work. Fearing (Fe 66) applied Foldy and Walecka's (Fo 64) giant dipole resonance model to RMC, and more recently Rood et al (Ro 74) considered the effects of including Coulomb and strong nuclear potentials. The advent of the "meson factories" has opened up the possibility of studying RMC on very light nuclei, for which newer calculations also exist (Hw 78A, Hw 78B).

Before this investigation was started and prior to the experiment of Hart et al (Ha 77), a serious discrepancy had existed between the experimentally measured and theoretically predicted values of the photon asymmetry for ^{40}Ca. Di Lella et al (Di 71) had determined the asymmetry to be $\bar{\alpha} \leq -0.32 \pm 0.48$, averaged over the interval $57 < k < 75$ Mev. This limit on α was at variance with the standard theory prediction $\bar{\alpha} = +0.75$, averaged over the appropriate photon energy interval. The sign discrepancy has now been largely resolved by the recent measurement of Hart et al (Ha 77) yielding $\bar{\alpha} = +0.90 \pm 0.50$, though there is still room for more precise numerical values.

In 1975, Fearing (Fe 75) proved that in the standard theory, assuming $g_s = 0$, the entire first order correction to α (in zeroth order, $\alpha = 1$) comes from $O(1/m^2)$ terms in the square of the transition matrix element. Since in previous calculations these $O(1/m^2)$ terms had not been retained consistently, the present study was undertaken with the primary objective of calculating the RMC photon spectrum and asymmetry consistently to $O(1/m^2)$. At the same time, such

theoretical improvements and supplementary investigations as proved feasible were also undertaken in an effort to explain the above discrepancy. Thus the standard theory calculation was carefully checked and extended to include all $O(1/m^2)$ terms in the square of the matrix element, the giant dipole resonance model calculation of Fearing was re-worked and updated, and a modification of the usual closure approximation involving nuclear sum rules was applied to RMC in order to gain some insight into the average maximum photon energy parameter k_m.

The calculation was done in impulse approximation, starting with the usual four-fermion interaction Hamiltonian describing ordinary muon capture on a free proton. The radiation field was minimally coupled to the hadron covariant portion of the Hamiltonian, which was reduced to non-relativistic form via the Foldy-Wouthuysen procedure, keeping all $O(1/m^2)$ terms. Transition amplitudes corresponding to a set of Feynman diagrams were obtained by pairing parts of the non-relativistically reduced hadron covariant with appropriate lepton covariants. The lepton covariants were reduced to non-relativistic form by neglecting the small components of the muon wave function. The resulting transition amplitudes were squared analytically and finally, upon choosing nuclear wave functions, observables were computed.

The detailed presentation of our work parallels the steps in the method of calculation just outlined. Chapter II

describes the relativistic Hamiltonian and its reduction to non-relativistic form. The lepton covariants required to yield amplitudes equivalent to those obtained from standard theory Feynman diagrams are enumerated. Chapter III deals with the problem of squaring the total transition amplitude. In several phases of this operation, symmetries were invoked to keep the number of terms at a manageable level. In Chapter IV the nuclear physics aspect of RMC is addressed. Reviews of the closure-harmonic oscillator and giant dipole resonance models are combined with descriptions of our improvements to them. In particular, Hartree-Fock wave functions are introduced and recent photoabsorption data utilized in the giant dipole resonance model. Chapter V contains our results for the ^{40}Ca photon spectrum and asymmetry for a variety of nuclear models. Chapter VI is devoted to describing a modified closure calculation of RMC. This approach involves nuclear sum rules and offers a possible means of determining the average maximum photon energy parameter k_m. Finally our conclusions are presented in Chapter VII, along with a few observations and suggestions regarding further work. Solely technical matters and discussions of peripheral issues have been relegated to the appendices.

II.

EFFECTIVE HAMILTONIAN FOR RADIATIVE MUON CAPTURE BY A NUCLEUS

A. Relativistic Hamiltonian

The impulse approximation was used to describe nuclear radiative muon capture (RMC). This approach excludes explicit nuclear mesonic degrees of freedom from consideration, the nucleus being regarded as a collection of nucleons which participate individually in the RMC process. Hence to calculate observables, such as the photon spectrum and asymmetry, it is first necessary to calculate the transition operator for radiative muon capture by a single proton. This was done by making a non-relativistic reduction of the relativistic Hamiltonian for radiative muon capture by a proton to obtain an effective Hamiltonian for this elementary process. The effective Hamiltonian for the nuclear case was subsequently obtained by summing the elementary effective Hamiltonian over all protons in the parent nucleus. Correction terms of order 1/A (A the atomic number) arising from nuclear center of mass motion (Kr 74) are expected to be small for ^{40}Ca and were omitted.

The Hamiltonian for a proton subject to strong, electromagnetic, and weak forces was taken to be : [1]

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[1] Notation and conventions are those of J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics (McGraw Hill Book Company, Inc., New York, 1964).

$$H = H_0 + H_E + H_W \qquad 2\text{-}1$$

where :

$$H_0 = \gamma_0 m + \gamma_0 \vec{\gamma}\cdot\vec{P} + V_0 + \gamma_0 V$$

$$H_E = e J_P (A_0 - \gamma_0 \vec{\gamma}\cdot\vec{A}) + \tfrac{1}{2}(X_N J_N + X_P J_P)\gamma_0 \sigma_{\mu\nu} F^{\mu\nu}$$

$$H_W = \frac{G}{\sqrt{2}} J_- \Big(g_V \gamma^\alpha + i\frac{g_M}{2m}\sigma^{\alpha\beta} q_\beta + \frac{g_S}{m_\mu} q^\alpha + g_A \gamma^\alpha \gamma_5 + \frac{g_P}{m_\mu} q^\alpha \gamma_5 + i\frac{g_T}{2m}\sigma^{\alpha\beta} q_\beta \gamma_5\Big)\mathcal{L}_\alpha \qquad 2\text{-}2$$

The strong interaction appears in H_0 as $V_0 + \gamma_0 V$, a combination scalar potential and one transforming as the zeroth component of a 4-vector. m is the nucleon mass, and $\vec{P}$ is the momentum operator. The form of the weak Hamiltonian H_W is the usual one evolved from the current x current theory of weak interactions. G is the Fermi constant, m_μ is the muon mass, and g_V, g_A, g_M, g_P, g_S, g_T are the weak couplings. The isospin operator J_- changes a proton into a neutron. $q_\alpha = n_\alpha - p_\alpha$ is the 4-momentum transferred to the hadronic current in the weak interaction. The leptonic current $\mathcal{L}_\alpha$ includes all interactions not directly coupled to the hadronic current, and its explicit form depends upon which particle radiates. Detailed expressions for $\mathcal{L}_\alpha$ are presented later in the chapter when we make the connection between our Hamiltonian formalism and the set of Feynman diagrams calculated in the standard theory. The electromagnetic Hamiltonian H_E contains Coulomb and radiation fields, including those due to the anomalous magnetic moments of proton and neutron. The electronic

charge is e, A_0 is the Coulomb field of the nucleus, and $\vec{A}$ is the photon field. χ_P=1.79, χ_N=-1.91 are the anomalous magnetic moments of proton and neutron in units of nuclear magnetons.

$F_{\mu\nu}$ are the electromagnetic field strengths. The isospin operators $\mathcal{T}_P, \mathcal{T}_N$ select protons and neutrons, respectively.

B. Non-Relativistic Reduction

The fully relativistic Hamiltonian H was reduced to a non-relativistic Hamiltonian using the Foldy-Wouthuysen transformation (Fo 50, Bj 64). The same non-relativistic Hamiltonian can be obtained from a Pauli reduction but, as Friar has pointed out (Fr 75) one must be careful to preserve the overall normalization of the transition rate and deal with initial and final states in a symmetric way to preserve Hermiticity. When working to $O(1/m^2)$, as here, or to higher orders, the Foldy-Wouthuysen procedure is preferable because algebraic complications are at a minimum. Furthermore the proper overall normalization is assured since to $O(1/m^2)$ the difference in normalization of relativistic and non-relativistic wave functions (Bj 64) is exactly cancelled by the difference in relativistic and non-relativistic phase space factors (Fe 78). The Foldy-Wouthuysen transformation decouples the relativistic Hamiltonian into two two-component Hamiltonians: one describes nucleon states and the other describes antinucleon states. Since the transformation reduces the Hamiltonian to

only so-called "even" operators which do not mix upper and lower components, an initial nucleon state is coupled only to a final nucleon state. The influence of the antinucleon states is manifest in the complicated form of the effective Hamiltonian.

The initial step in the reduction involved decomposing H into odd and even operators, which respectively do and do not mix upper and lower components of the nucleon wave function. Thus :

$$H \equiv \gamma_0 m + \overbrace{\mathcal{E}_0 + \mathcal{E}_W}^{\mathcal{E}} + \overbrace{\mathcal{O}_0 + \mathcal{O}_W}^{\mathcal{O}} \tag{2-3}$$

where :

$$\begin{aligned}
\mathcal{E}_0 &= e\,\mathcal{J}_P A_0 + \gamma_0 V + V_0 + (\chi_N \mathcal{J}_N + \chi_P \mathcal{J}_P)\,\gamma_0\,\sigma^{ij} F^{ij} \\
\mathcal{O}_0 &= \gamma_0\,\vec{\gamma}\cdot(\vec{P} - e\,\mathcal{J}_P \vec{A}) + 2i\,(\chi_N \mathcal{J}_N + \chi_P \mathcal{J}_P)\left(\vec{\gamma}\cdot(\vec{\nabla}A_0) + \vec{\gamma}\cdot\dot{\vec{A}}\right) \\
\mathcal{E}_W &= \frac{G}{\sqrt{2}}\,\mathcal{J}_-\left(i\,\frac{g_M}{2m}\,\sigma^{\alpha\beta} q_\beta + \frac{g_S}{m_\mu}\, q^\alpha + g_A\,\gamma^\alpha \gamma_5\right)\mathcal{L}_\alpha \\
\mathcal{O}_W &= \frac{G}{\sqrt{2}}\,\mathcal{J}_-\left(g_V\,\gamma^\alpha + \frac{g_P}{m_\mu}\, q^\alpha \gamma_5 + i\,\frac{g_T}{2m}\,\sigma^{\alpha\beta} q_\beta\,\gamma^5\right)\mathcal{L}_\alpha
\end{aligned} \tag{2-4}$$

To enable a consistent calculation of RMC observables to appropriate orders in the expansion parameter 1/m, H was transformed so that the odd operators appeared in lowest order as $O(1/m^3)$ terms :

$$H'' = \gamma_0\left(m + \frac{\mathcal{O}^2}{2m}\right) + \mathcal{E} - \frac{1}{8m^2}\left[\mathcal{O},[\mathcal{O},\mathcal{E}]\right] - \frac{i}{8m^2}\left[\mathcal{O},\dot{\mathcal{O}}\right] \tag{2-5}$$

The explicit form of the non-relativistic Hamiltonian was determined by a combined computer and hand evaluation. A FORTRAN program performed the initial operator algebra,

which amounted to commuting operators into a standard form. Each vector space (isospin, Dirac, etc.) having elements in the expression for H" was represented by a linked list structure composed of those elements. The operator commutation rules appropriate to each vector space were coded and then utilized as the commutators were evaluated. The basic algorithm required the product of two terms to be put into a predetermined canonical form which subsequently was either printed out or used in further evaluations. The result was an expression for H" involving Dirac matrices which were expressed in terms of (two-component) Pauli matrices. The final simplifications and projection of the nucleon states (upper components) were done by hand.

Throughout the procedure outlined above, the exact expressions for the lepton covariants $\mathcal{L}_\alpha$ were left unspecified. This generality was in keeping with the as yet quite general form for the effective Hamiltonian, which describes several different weak, electromagnetic, and strong processes. The next task was to isolate those parts of the effective Hamiltonian which specifically describe radiative muon capture.

C. Effective Hamiltonian

To begin constructing an effective interaction Hamiltonian for radiative muon capture, the non-relativistic Hamiltonian H" was written as :

$$H'' = H_o'' + H_I'' \qquad 2\text{-}6$$

where H_o'' is the kinetic energy plus strong potential part of H'' and H_I'' is composed of terms involving the weak and electromagnetic interactions. The states, Green's functions, etc. corresponding to H_o'' are labelled with the subscript o. The calculation of quantities corresponding to the full Hamiltonian H'' then proceeded using the quantities corresponding to H_o'' as a starting point (Ne 66). Thus the full nucleon Green's function is :

$$\begin{aligned} G^{\pm}(E) &= G_o^{\pm}(E)\, H_I''\, G^{\pm}(E) + G_o^{\pm}(E) \\ &= G_o^{\pm}(E) + G^{\pm}(E)\, H_I''\, G_o^{\pm}(E) \end{aligned} \qquad 2\text{-}7$$

The superscripts plus and minus denote retarded and advanced Green's functions, respectively. For the transition from an initial state i to a final state f, the full T-matrix is :

$$T_{fi}(E) = T_{o_{fi}}(E) + \left(\varphi_o^{(-)}(E,f), \left[H_I'' + H_I''\, G^{+}(E)\, H_I''\right] \varphi_o^{(+)}(E,i)\right) \qquad 2\text{-}8$$

The photon emission and weak interaction processes appearing in H_I'' were calculated to first order in perturbation theory. This is a good approximation since the (dimensionless) expansion parameters $e^2/\hbar c$ and :

$$\frac{g^2}{\hbar c} = \frac{1}{\sqrt{2}} \cdot \frac{1}{4\pi} \cdot \frac{1}{\hbar c} \left(\frac{m_W c}{\hbar}\right)^2 G \simeq 6.5 \times 10^{-11} (m_W c^2)^2$$

with $m_W c^2$ the intermediate vector boson mass in Mev and G the Fermi constant, are both < 0.01. Thus $G^{+}(E)$ was replaced by $G_o^{+}(E)$ and only those terms in T_{fi} in which e and G each appeared just once were retained. The leading term $T_{o_{fi}}$

represents scattering by the nuclear potential and can be dropped. The radiative muon capture T-matrix thus takes the form :

$$T_{fi}^{RMC}(E) = (\varphi_0^{(-)}(E,f), [H_I'' + H_I'' G_0^+(E) H_I''] \varphi_0^{(+)}(E,i)) \qquad 2\text{-}9$$

It is completely specified when the lepton covariants appearing in H_I'' are specified.

The initial effort was directed towards selecting the Hamiltonian corresponding to the Feynman diagrams shown in fig. 1. The calculation of these diagrams comprises the standard theory of RMC (Ro 65), so quite naturally was chosen as the starting point for our investigation of higher order terms in the effective Hamiltonian. In this approach the nuclear potential is omitted, hence the propagator $G_0^+(E)$ appearing in Eq. 2-9 becomes a free particle propagator. Each of the $\mathcal{L}_\alpha$ described in the following list was associated with a corresponding Feynman diagram as follows :

1. incoming muon, weak vertex, outgoing neutrino. This covariant was associated with a nucleon radiating, fig. 1(b) and 1(c).

$$\mathcal{L}_\alpha(x) = \bar{\varphi}_\nu(x)\, \gamma_\alpha (1-\gamma_5)\, \varphi_\mu(x)$$

$$= \exp(i(\nu-\mu)\cdot x)\, \bar{u}_\nu\, \gamma_\alpha (1-\gamma_5)\, u_\mu \qquad 2\text{-}10$$

2. incoming muon which radiates, weak vertex, outgoing neutrino. This covariant was associated with the non-radiating part of the nucleon current, fig. 1(a).

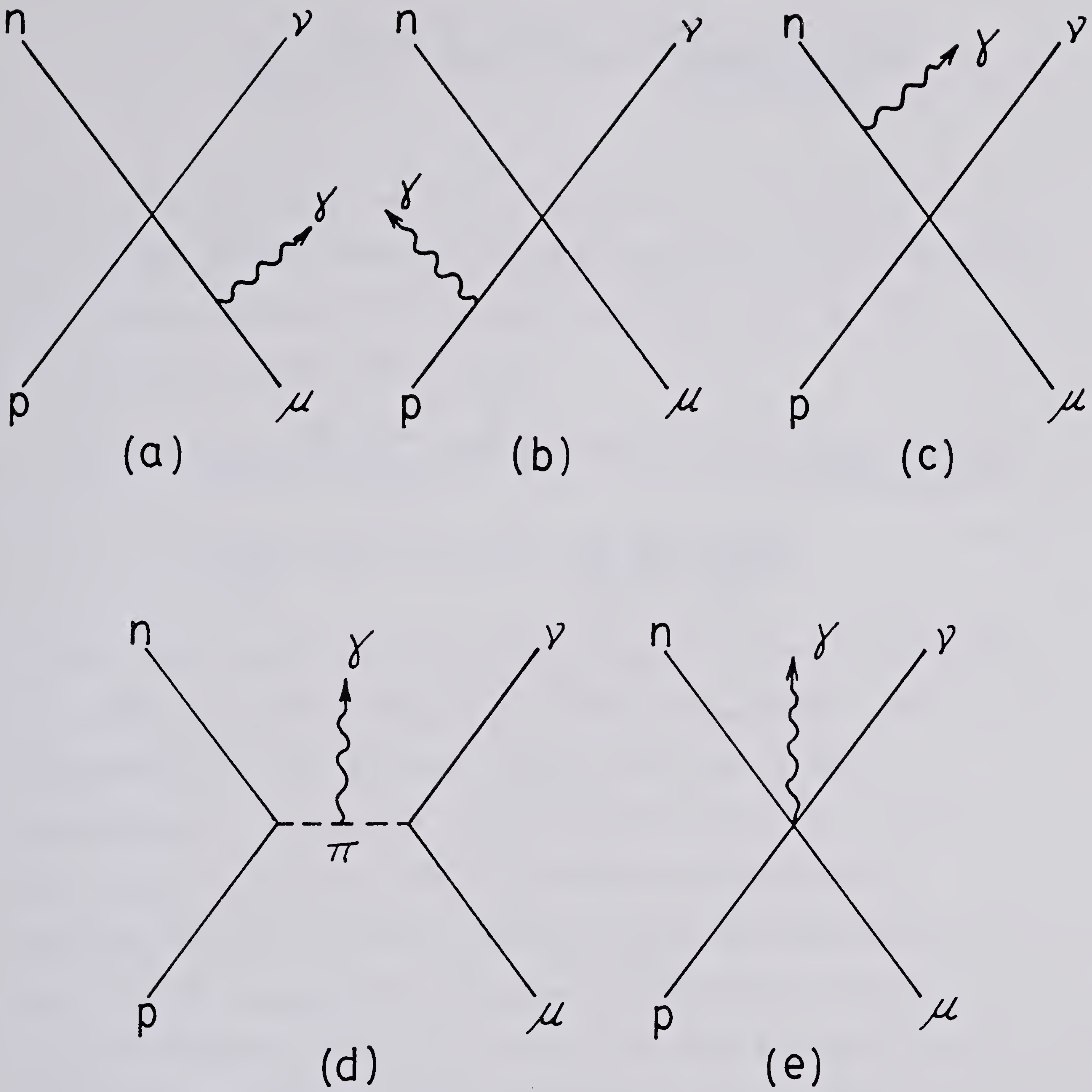

Fig. 1. Diagrams contributing to radiative muon capture in the standard theory. Diagrams (b) and (c) include radiation from the anomalous magnetic moment of the proton and neutron.

$$\mathcal{L}_\alpha(x) = \int d^4y\, \bar{\Psi}_\nu(x)\, \gamma_\alpha(1-\gamma_5)\, i S_F(x-y)\, ie \not{A}(y)\, \Psi_\mu(y)$$

$$= -\frac{e}{\sqrt{2k}} \exp(i(\nu+k-\mu)\cdot x)\, \bar{u}_\nu\, \gamma_\alpha(1-\gamma_5) \frac{(\not{\mu}-\not{k}+m_\mu)\not{\epsilon}}{(\mu-k)^2 - m_\mu^2} u_\mu \qquad 2\text{-}11$$

3. incoming muon, weak vertex coupling to the nucleons via a pion which radiates, outgoing neutrino. This covariant was associated with the non-radiating part of the nucleon current, fig. 1(d).

$$\mathcal{L}_\alpha(x) = -\frac{2em_\mu}{\sqrt{2k}} \iint \frac{d^4y\, d^4z}{(2\pi)^4} \exp\big(i(k-z)\cdot x + i(\nu+z-\mu)\cdot y\big) \frac{\bar{u}_\nu\, \epsilon_\alpha(1+\gamma_5)\, u_\mu}{z^2 - m_\pi^2}$$

$$= -\frac{2em_\mu}{\sqrt{2k}} \exp(i(\nu+k-\mu)\cdot x) \frac{\bar{u}_\nu\, \epsilon_\alpha(1+\gamma_5)\, u_\mu}{(\mu-\nu)^2 - m_\pi^2} \qquad 2\text{-}12$$

Figure 1(e) comes from the requirement of gauge invariance, ie. from the substitution $\vec{p}^{\,\alpha} \rightarrow \vec{p}^{\,\alpha} - eA^\alpha$ in H_W in Eq. 2-2. Amplitudes for the Feynman diagrams in fig. 1 were constructed by substituting the above covariants into Eq. 2-9, which represents the effective non-relativistic transition matrix element obtained from the Hamiltonian in Eq. 2-4. In simplifying the lepton covariants, it was assumed that the muon was at rest and that the muon wave function was constant over the nuclear volume. The prescription described by Luyten et al (Lu 63) was used to obtain an appropriate average value for the muon wave function.

Specification of the lepton covariants completed the construction of an $O(1/m^2)$ effective Hamiltonian which describes radiative muon capture by a proton. Performing a

sum over the constituent protons of the capturing nucleus then yielded a Hamiltonian for nuclear radiative muon capture of the form :

$$H^{eff} = \chi_\nu^+ (1 - \vec{\sigma}_L \cdot \hat{\nu}) \mathcal{M} \chi_\mu \qquad 2\text{-}13$$

with $\vec{\sigma}_L$ the lepton spin matrices and the nuclear operators contained in :

$$\mathcal{M} = \mathcal{M}_1 + \vec{\sigma}_L \cdot \vec{\mathcal{M}}_2 \qquad 2\text{-}14$$

To facilitate manipulation by the algebraic squaring routines, $\mathcal{M}$ was arranged into the form :

$$\mathcal{M} = \sum_{j=1}^{Z} \mathcal{T}(j) \left\{ \sum_{i=1}^{N} (E_i + D_i + C_i) F_i + \sum_{i=N+1}^{M} (E_i + D_i + C_i) \vec{\sigma}_L \cdot \vec{G}_i \right\}_j e^{-i \vec{S} \cdot \vec{r}_j} \qquad 2\text{-}15$$

where F_i and $\vec{G}_i$ are functions of the vectors $\vec{\epsilon}$, $\vec{k}$, $\vec{\nu}$, $\vec{P}/m$, and $\vec{\sigma}$. The coefficients E_i, D_i, and C_i correspond to terms of order 1, 1/m, and $1/m^2$ respectively. The complete Hamiltonian appears in Appendix B. The connection with the work of Rood and Tolhoek (Ro 65) is made by taking the contact approximation for the nucleon propagator $G_0^+(E)$, ie. letting the mass of the propagating nucleon go to infinity. When this is done, agreement is found with their effective Hamiltonian which contains only a selected few $O(1/m^2)$ terms. Other parts of H_I'' are found to be in agreement with previous Foldy-Wouthuysen reductions of weak (Fr 66, Oh 66) and electromagnetic (Mc 62) Hamiltonians. [2]

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[2] The last term in the expression for G_A in Eq. (4) of (Fr 66) is dimensionally incorrect. The factor $(\omega - k^2/2m)$ should read $(\omega/k - k/2m)$.

An extension of the standard theory which includes Coulomb and strong nuclear potentials has been considered by Rood et al (Ro 74), who use the Martin and Glauber formalism (Ma 58). Much of their work involves wholly numerical evaluations of RMC amplitudes which are checked against the results of Rood and Tolhoek (Ro 65) in the limit of vanishing nuclear potentials. The original intent of our work was to calculate, in an analytic fashion, higher order corrections to the photon spectrum and asymmetry, and this was accomplished for the standard theory. The task of modifying our existing calculation to include Coulomb and strong nuclear potentials appeared to require considerable further effort, however, and was not undertaken. Some further discussion of the role of exterior potentials in RMC appears in the presentation of results in Chapter V.

III.

THE TRANSITION MATRIX ELEMENT SQUARED

A. Squaring Strategy

The calculation of observables for radiative muon capture requires that the transition matrix element be squared and summed on appropriate spins. The steps in the derivation are the usual ones encountered in squaring Feynman amplitudes (Bj 64). With the inclusion of phase space and kinematic factors, the probability for radiative muon capture corresponding to the nuclear transition $|a\rangle \rightarrow |b\rangle$, the photon and neutrino being emitted with momenta $\vec{k}$ and $\vec{\nu}$, while the polarizations of the initial muon and photon are specified by $\vec{S}$ and λ is (Ro 65) :

$$P_{ab}(\vec{k},\vec{\nu},\lambda,\vec{S})\,dk\,d\Omega_k\,d\Omega_\nu = \frac{\alpha}{4(2\pi)^4}\,|\varphi_\mu|^2_{av}\sum_{\bar{a}b} k(k_m^{ab}-k)^2\,|M_{ba}|^2\,dk\,d\Omega_k\,d\Omega_\nu \tag{3-1}$$

An average value $|\varphi_\mu|^2_{av}$ of the muon wave function over the nuclear volume has been extracted from the matrix element M_{ba} $=\langle b|H^{eff}|a\rangle$. $\alpha=1/137$ is the fine structure constant. Practical difficulties encountered in the present calulation of $|M_{ba}|^2$ arose as a result of the large number of terms considered, which are sufficiently numerous to warrant being manipulated by a computer code. For this reason the algebraic language REDUCE2 (He 73) was used to do a large part of the algebra. In the current chapter the methodology employed in obtaining the squared matrix element is described in detail.

B. Lepton Spin Algebra

Beginning with the effective Hamiltonian :

$$H^{eff} = \chi_\nu^\dagger (1 - \vec{\sigma}_L \cdot \hat{\nu}) \mathcal{M} \chi_\mu \tag{3-2}$$

the square of the matrix element was constructed, taking account of the initial muon polarization by introducing the spin projection operator $(1+\vec{\xi}\cdot\vec{\sigma}_L)/2$ to the immediate left of the muon spinor :

$$\sum_{spins} |M_{ba}|^2 = \frac{1}{4} \sum_{spins} |\chi_\nu^\dagger (1 - \vec{\sigma}_L \cdot \hat{\nu}) \mathcal{M} (1 + \vec{\sigma}_L \cdot \vec{\xi}) \chi_\mu|^2 \tag{3-3}$$

The most general scalar form for $\mathcal{M}$ in terms of the lepton spin matrices $\vec{\sigma}_L$ is :

$$\mathcal{M} = \mathcal{M}_1 + \vec{\sigma}_L \cdot \vec{\mathcal{M}}_2 \tag{3-4}$$

Substituting this form in Eq. 3-3 and doing the trace algebra yields :

$$\sum_{spins} |M_{ba}|^2 = 2\{(1-\hat{\nu}\cdot\hat{\xi})\mathcal{M}_1\mathcal{M}_1^\dagger + (1+\hat{\nu}\cdot\hat{\xi})\vec{\mathcal{M}}_2\cdot\vec{\mathcal{M}}_2^\dagger - i(\hat{\nu}+\hat{\xi})\cdot\vec{\mathcal{M}}_2\times\vec{\mathcal{M}}_2^\dagger - 2\,\mathrm{Re}\,[\mathcal{M}_1(\hat{\nu}-\hat{\xi}+i\hat{\nu}\times\hat{\xi})\cdot\vec{\mathcal{M}}_2^\dagger + \hat{\nu}\cdot\vec{\mathcal{M}}_2\;\hat{\xi}\cdot\vec{\mathcal{M}}_2^\dagger]\} \tag{3-5}$$

The expressions $\mathcal{M}_1$ and $\vec{\mathcal{M}}_2$ are given explicitly in Appendix B. Eq. 3-5 was evaluated analytically with the help of the computer language REDUCE2. Before describing the explicit evaluation though, some general simplifications of the squared matrix element are discussed.

C. Symmetries in the Squared Matrix Element

The squared matrix element exhibits particular symmetries which can be utilized to simplify it considerably. The manifestation of these symmetries may require some idealizing assumptions, and where this is the case, specific mention is made of these.

Nucleon Spin

Consider that part of the squared matrix element which, in closure approximation, has the form :

$$|\mathcal{M}|^2 = \langle a| \sum_j O^{+}(j) \sum_k O'(k)\, \vec{\sigma}_\alpha(k) |a\rangle \qquad 3\text{-}6$$

where $O^{+}(j)$ and $O'(k)$ are single particle operators independent of spin. It is assumed that

1. $|a\rangle$ is a Slater determinant of single particle wave functions including spin-orbit coupling and that
2. $|a\rangle$ has closed subshells.

A straightforward calculation using the techniques of angular momentum algebra (Ed 57) shows that the one-body ($j=k$) part of $|\mathcal{M}|^2$ is identically zero and that its two-body ($j \neq k$) part is proportional to the product of 3-j symbols

$$\begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & l' & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

which also vanishes. Here l_1 and l_2 are orbital angular momenta of the single-particle states. Hence terms in our squared matrix element involving only one nucleon $\vec{\sigma}$ are identically zero.

In the case where closure is not used, Luyten et al (Lu 63) find that in addition to the above assumptions one requires no spin-orbit coupling in order that $\langle 1\rangle\langle\vec{\sigma}\rangle=0$.

<u>Integration on Neutrino Angle</u>

When the initial muon has polarization $\vec{\zeta}$, the available vectors which can be used to construct observables after integrating over neutrino angle and summing on spins are $\vec{\zeta}$ and $\vec{k}$. As a result the differential photon spectrum exhibits the following dependence on $\vec{\zeta}$ and $\vec{k}$:

$$\frac{d\sigma}{d\Omega_k} \sim \int d\Omega_\nu |M_{ba}|^2 \sim 1 + \alpha\, \hat{\zeta}\cdot\hat{k} \qquad 3\text{-}7$$

with α the photon asymmetry. Hence $\frac{d\sigma}{d\Omega_k}$ is invariant under the transformation $\vec{\zeta},\vec{k}\rightarrow -\vec{\zeta},-\vec{k}$. If the final and initial nuclear states have definite parity, the transformation $\vec{r}\rightarrow-\vec{r}$ applied to the matrix element will not affect the wave functions appearing in $|M_{ba}|^2$ since each state appears twice, however $\vec{P}$ is is changed to $-\vec{P}$. Making these transformations on $|M_{ba}|^2$ and now taking $\vec{\nu}\rightarrow-\vec{\nu}$ has the effect of making each term in the matrix element an eigenstate of the transformation operator which changes $\vec{\nu}\rightarrow-\vec{\nu}$ and has eigenvalue +1 or -1. Thus from an operational point of view, all terms exhibiting odd behaviour under the transformation $\vec{\nu},\vec{k},\vec{\zeta},\vec{P}\rightarrow -\vec{\nu},-\vec{k},-\vec{\zeta},-\vec{P}$ vanish when integrated over neutrino solid angle. Note that as a consequence of this, the final result for the squared matrix element integrated over neutrino angle will not contain any vector triple products.

Relations Between Squared Nuclear Matrix Elements

Define the following set of squared nuclear matrix elements for ordinary muon capture :

$$|M_\beta|^2 = \sum_{\bar{a}b} \left(\frac{\nu_{ab}}{m_\mu}\right)^2 \int \frac{d\Omega_\nu}{4\pi} \, |\langle b| \sum_{i=1}^{A} O_\beta(i) \exp(-i\vec{\nu}_{ab}\cdot\vec{r}_i)|a\rangle|^2 \tag{3-8}$$

where β can be V, A, or P with :

$$O_V(i) = 1(i) \,, \qquad O_A(i) = \frac{1}{\sqrt{3}}\,\vec{\sigma}(i) \,, \qquad O_P(i) = \hat{\nu}\cdot\vec{\sigma}(i) \tag{3-9}$$

Then the following relations between matrix elements (Fo 64) :

$$|M_V|^2 = |M_A|^2 = |M_P|^2 \tag{3-10}$$

hold exactly in a single particle model without spin-orbit coupling, and hold to a good approximation in cases where the spin dependence of the inter-nucleon forces is fairly weak (Lu 63, Fo 64, Go 74). Applied to radiative muon capture (Ro 65, Fe 66), the relations appear as :

$$\begin{aligned}
&\sum_{\bar{a}b} (k_m^{ab}-k)^2 |\langle b|\sum_i \tau_-(i)\exp(-i\vec{S}_{ab}\cdot\vec{r}_i)\,\vec{\sigma}(i)\,|a\rangle|^2 \\
&\qquad = 3\sum_{\bar{a}b} (k_m^{ab}-k)^2 |\langle b|\sum_i \tau_-(i)\exp(-i\vec{S}_{ab}\cdot\vec{r}_i)|a\rangle|^2 \\
&\sum_{\bar{a}b} (k_m^{ab}-k)^2 \vec{A}\cdot\langle b|\sum_i \tau_-(i)\exp(-i\vec{S}_{ab}\cdot\vec{r}_i)\vec{\sigma}(i)|a\rangle\, \vec{B}^*\cdot\langle b|\sum_i \tau_-(i)\exp(-i\vec{S}_{ab}\cdot\vec{r}_i)\vec{\sigma}(i)|a\rangle \\
&\qquad = \sum_{\bar{a}b} \vec{A}\cdot\vec{B}^*(k_m^{ab}-k)^2 |\langle b|\sum_i \tau_-(i)\exp(-i\vec{S}_{ab}\cdot\vec{r}_i)|a\rangle|^2
\end{aligned} \tag{3-11}$$

where $\vec{s}_{ab} = (\vec{k}+\vec{\nu})_{ab}$, and $\vec{A}$, $\vec{B}$ are arbitrary vectors. Note that the net effect of the relations 3-10 is to remove the nucleon spin operators from the matrix element.

D. Evaluation of the Square of the Matrix Element

With the help of the simplifying relations detailed in the previous section, the analytic evaluation of Eq. 3-5 was carried out. The algebraic manipulations were done in three distinct parts, each part being accomplished by a REDUCE2 program written specifically for the task.

The initial phase of the work involved classifying the terms in the effective Hamiltonian according to :

1. their behaviour under the transformation $\vec{\nu}, \vec{k}, \vec{\xi}, \vec{P} \rightarrow -\vec{\nu}, -\vec{k}, -\vec{\xi}, -\vec{P}$, and
2. the number of nucleon $\vec{\sigma}$'s appearing in each term.

All possible forms emerging from this classification scheme (in which the Coulomb gauge was used and the photon polarization vector $\vec{\epsilon}^{(\lambda)} = \frac{(\hat{i} - i\lambda\hat{j})}{\sqrt{2}}$ was made to appear explicitly) were enumerated and substituted into Eq. 3-4. A list of these forms can be found in Appendix B. Eq. 3-5 was then evaluated and the simplifications of the preceding section associated with points 1. and 2. above were done. Each product of forms in this intermediate expression represented a large number of terms in the squared matrix element. Elimination of the photon polarization vector was effected by the relation :

$$\epsilon_i^{(\lambda)} \epsilon_j^{(\lambda)} = \frac{1}{2} \left[\delta_{ij} - \hat{k}_i \hat{k}_j + i \lambda \epsilon_{ijk} \hat{k}_k \right] \qquad 3\text{-}12$$

which follows directly from the explicit expression for $\vec{\epsilon}^{(\lambda)}$. This first stage of the squaring procedure was carried out by the program SQUARE1.

The next step was to substitute the original terms back into the expression for the squared matrix element, which at this stage was simplified insofar as the symmetry properties of the forms had been fully exploited. The re-substitutions could not be done all at once as the number of terms generated by the substitutions far exceeded REDUCE2's storage capacity. Rather the squared matrix element was sub-divided and the segments processed one at a time. The program SQUARE2 handled the re-substitutions and the elimination of nucleon $\vec{\sigma}$'s via the relations in Eqs. 3-11a and 3-11b.

The final analytic form of the squared matrix element was produced by the program SQUARE3. An integration over the (experimentally unobserved) neutrino solid angle was performed, introducing $y=\hat{k}\cdot\hat{\nu}$ as an integration variable. The integrals of various powers of y multiplied by a squared nuclear matrix element were systematically labelled. A feature of the REDUCE2 language permitted output of the squared matrix element from SQUARE3 in FORTRAN, thus greatly facilitating further numerical work.

In our final expression for the squared matrix element all terms of $O(1/m^2)$ and the $O(1/m^3)$ terms coming from the product of $O(1/m)$ and $O(1/m^2)$ terms in the effective Hamiltonian were retained. These included the so-called "velocity terms", ie. those terms involving nucleon momenta. Appendix C contains an explicit expression for the squared matrix element which is fully specified up to the nuclear

matrix elements. A number of nuclear models permitting evaluation of the latter quantities is presented in the next chapter.

IV.

NUCLEAR MODELS FOR RADIATIVE MUON CAPTURE

A. RMC Models for Complex Nuclei

In this chapter two nuclear models for radiative muon capture and our improvements to them are discussed. The standard closure-harmonic oscillator (CHO) model employed by Rood and Tolhoek (Ro 65) and the giant dipole resonance (GDR) model of Foldy and Walecka (Fo 64) as applied to radiative muon capture by Fearing (Fe 66) are reviewed. Included in the discussion of the harmonic-oscillator model is an improvement to it in the form of a more sophisticated shell model (SBL) wave function due to Shao, Bassichis and Lomon (Sh 72). In addition, a variant of the closure-harmonic oscillator model involving nuclear sum rules is mentioned briefly. A more complete discussion of this latter approach as applied to radiative muon capture is contained in Chapter VI. The standard closure-harmonic oscillator model is discussed first.

B. Closure - Harmonic Oscillator Model

<u>Simple Wave Functions</u>

Recalling Eq. 3-1, which gives the probability for radiative muon capture corresponding to the nuclear transition $|a\rangle \rightarrow |b\rangle$:

$$P_{ab}(\vec{k},\vec{\nu},\lambda,\vec{s})\, dk\, d\Omega_k\, d\Omega_\nu = \frac{\alpha}{4(2\pi)^4} |\varphi_\mu|^2_{av} \sum_{\bar{a}b} k(k_m^{ab}-k)^2 |M_{ba}|^2 dk\, d\Omega_k\, d\Omega_\nu \qquad 3\text{-}1$$

where $M_{ba}=\langle b|H^{eff}|a\rangle$, it is apparent that the exact calculation of :

$$\sum_{\bar{a}b} (k_m^{ab}-k)^2 |M_{ba}|^2$$

requires a knowledge of the wave functions of the accessible states in the daughter nucleus. For ^{40}Ca, the maximum energy available to excite the daughter nucleus ^{40}K is :

$$E_m = m_\mu - E_{BE} - (m_N - m_P) + E_c \simeq 110.4 \text{ MeV} \qquad 4\text{-}1$$

in which E_{BE} is the muon binding energy and E_c is the Coulomb energy difference between initial and final nuclear states. Since it is expected that most of the transition strength lies in transitions to relatively low-lying states in the daughter nucleus, the use of the closure approximation is appropriate (Pr 59). Closure involves replacing the sum on all possible final states $|b\rangle$ in Eq. 3-1 by a sum on a complete set of states :

$$\sum_b |b\rangle\langle b| = 1 \qquad 4\text{-}2$$

The introduction of superfluous states $E>E_m$ in this manner is not a cause for concern if the majority of transitions are to states of sufficiently low energy, while the advantage gained is that specific final state dependent quantities are replaced by suitable averages. Thus k_m^{ab} is replaced by an average maximum photon energy k_m corresponding to an average exitation energy of the daughter nucleus :

$$k_m^{ab} = m_\mu - (m_N - m_P) - E_{BE} - (E_b - E_a) \rightarrow k_m \qquad 4\text{-}3$$

with the associated notational change :

$$(\vec{v}+\vec{k})_{ab} \rightarrow \vec{v}+\vec{k} \equiv \vec{s} \qquad 4\text{-}4$$

E_a, E_b are the energies of the initial and final nuclear states, respectively. It now remains to evaluate products of nuclear matrix elements of the general form :

$$\langle O_1\rangle\langle O_2\rangle^* = \langle a|\sum_{j,k} \tau_-(j)\tau_+(k)\exp(-i\vec{s}\cdot(\vec{r}_j-\vec{r}_k))\,O_1(j)\,O_2^\dagger(k)|a\rangle \qquad 4\text{-}5$$

which, since they are ground-state expectation values of two-body operators, can be calculated relatively easily in a nuclear model. The sums on j and k run over all occupied nucleon states of the parent nucleus. The operators O_1 and O_2 are in general functions of nucleon momenta and spins. Here $\langle 1\rangle\langle 1\rangle^*$ will be referred to as the main nuclear matrix element squared since it dominates numerically. For ^{40}Ca, the result for the main nuclear matrix element squared in the closure-harmonic oscillator model is :

$$\langle 1\rangle\langle 1\rangle^* = 20 - (20 + 2.5\eta^4 - 0.25\eta^6 + 0.03125\eta^8)\, e^{-\eta^2/2} \qquad 4\text{-}6$$

with $\eta^2 = b^2(\vec{s}\cdot\vec{s}) = (sb)^2$ and b the oscillator parameter. Explicit details of the evaluation of the products of the nuclear matrix elements required here can be found in Appendix D.

<u>Hartree-Fock Wave Functions</u>

A straightforward improvement to the closure-harmonic oscillator model was achieved by replacing the simple

single-particle oscillator wave functions for the nuclear matrix elements $\langle 1\rangle\langle 1\rangle^*$ and $\langle 1\rangle\langle P\rangle^*$ with the more realistic ones of Shao et al (Sh 72), calculated from the Feshbach-Lomon potential using a Hartree-Fock technique. [3] Since the SBL wave function includes spin-orbit coupling, the relations $|M_V|^2=|M_A|^2=|M_P|^2$ adapted for RMC (Eqs. 3-11) can be expected to hold only approximately. An explicit calculation shows $|M_A|^2=|M_V|^2$ to better than 3% and $|M_P|^2=|M_A|^2+\Delta$, where the correction Δ involves the tensor coupling between M_P and M_P^*. Δ is identically zero when $|M_P|^2$ is integrated on photon and neutrino solid angle, and is estimated to be very small otherwise. These results tend to indicate that the approximation $|M_V|^2=|M_A|^2=|M_P|^2$ is a good one, and we have used it here.

Shao and collaborators have expressed their wave functions in a basis of harmonic oscillator functions each having the same oscillator parameter, thus permitting the main nuclear matrix element squared to be calculated analytically. Working with a basis of oscillator functions meant that the actual form of the result for $\langle 1\rangle\langle 1\rangle^*$ was known, so that the problem became one of determining coefficients. This task was straightforward as the angular momentum algebra was not particularly complicated, and closed form expressions exist for the radial integrals

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[3] $\langle 1\rangle\langle P\rangle^*$ was expressed in terms of the two-body part of $\langle 1\rangle\langle 1\rangle^*$ and was calculated concurrently with it. The other nuclear matrix elements, being of lesser numerical significance, were calculated using the simple wave functions described in the previous section. See Appendix D for details.

(De 66). The large number of terms to be multiplied and summed dictated a computer evaluation. Details of the calculation are to be found in Appendix E. The result :

$$\langle 1\rangle\langle 1\rangle^* = 20-(20-3.05\eta^2+5.153\eta^4-1.627\eta^6+0.414\eta^8)e^{-\eta^2/2} \qquad 4\text{-}7$$

may not be compared directly with the result for simple wave functions given in Eq. 4-6 since in this latter equation b=2.03 fm is appropriate, while in Eq. 4-7 b=1.90 fm should be used. Different methods for fixing b have resulted in these two different values. Rood et al (Ro 65) determined b=2.03 fm. by requiring that the simple harmonic oscillator wave functions reproduce the ^{40}Ca r.m.s. charge radius obtained from electron scattering experiments. Shao et al (Sh 72) determined b=1.90 fm. by requiring that their Hartree-Fock wave functions yield optimum values for the ^{16}O r.m.s. charge radius and binding energy.

It may be noted that the expression in Eq. 4-7 contains a η^2 term whereas the expression in Eq. 4-6 does not. The explanation for this difference of form (Be 71) is to be found in the property of factorization of simple harmonic oscillator wave functions into products independently describing relative and center of mass motion, combined with the fact that for ^{40}Ca, N=Z. Explicitly, if isospin is a good quantum number, the closure result for the main nuclear matrix element squared can be expressed :

$$\langle 1\rangle\langle 1\rangle^* = \langle a|\sum_j \tau_+(j)\exp(i\vec{s}\cdot\vec{r}_j)\sum_i \tau_-(i)\exp(-i\vec{s}\cdot\vec{r}_i)|a\rangle$$

$$\langle 1\rangle\langle 1\rangle^{*} = \frac{1}{2}\langle a|\sum_j J_3(j)\exp(i\vec{s}\cdot\vec{r}_j)\sum_i J_3(i)\exp(-i\vec{s}\cdot\vec{r}_i)|a\rangle \qquad 4\text{-}8$$

For nuclei with N=Z the term of second order in $\eta = sb$ in Eq. 4-8 is :

$$-\frac{1}{2}\langle a|\sum_i J_3(i)\,\vec{s}\cdot\vec{r}_i \sum_j J_3(j)\,\vec{s}\cdot\vec{r}_j|a\rangle$$
$$= -\frac{1}{2}Z^2\langle a|\vec{s}\cdot(\vec{R}_P-\vec{R}_N)^2|a\rangle \qquad 4\text{-}9$$

where $\vec{R}_P$ and $\vec{R}_N$ are center of mass coordinates of the protons and neutrons. For the simple harmonic oscillator, the center of mass motion has a ground state wave function proportional to :

$$\exp\left(-\frac{Z}{4b^2}(\vec{R}_P-\vec{R}_N)^2\right)$$

so that the closure-harmonic oscillator model gives for Eq. 4-9 :

$$-\frac{1}{2}Z\eta^2$$

Since the general form of the result for $\langle 1\rangle\langle 1\rangle^{*}$ is :

$$\langle 1\rangle\langle 1\rangle^{*} = Z - Z\{1 + C_1\eta^2 + C_2\eta^4 + \ldots\}\exp(-\eta^2/2) \qquad 4\text{-}10$$

the term $Z(-\frac{1}{2}\eta^2)$ is entirely accounted for by the expansion of the exponential in Eq. 4-10, therefore implying $C_1=0$ for the closure-harmonic oscillator model. The SBL wave function on the other hand cannot be factored into terms which individually describe relative and center of mass motion, so that the evaluation of Eq. 4-9 will not in general lead to the result $Z(-\frac{1}{2}\eta^2)$, and hence C_1 will be non-zero.

A crucial shortcoming of the closure-harmonic oscillator model is that it predicts ordinary capture rates which are approximately a factor of two larger that those

measured for ^{16}O and ^{40}Ca. A significant part of the discrepancy may be understood once it is realized that most of the capture occurs through the first forbidden dipole matrix elements (Ti 49) to states lying in the region of excitation of the giant dipole resonance. The energies of these states should be accurately reproduced by the nuclear model because the capture rate depends strongly on the maximum photon energy (the radiative rate goes as k_m^6). In the simple shell model, the giant dipole resonance does emerge as a collective feature, but at too low an energy, so that for a simple shell model calculation which does not employ closure, the dipole contribution to the rate will be overestimated. Presumably the closure-harmonic oscillator model rate is overestimated for the same reason. A means of avoiding this difficulty was developed by Foldy and Walecka (Fo 64) in an investigation of ordinary muon capture. Their approach, subsequently referred to as the giant dipole resonance model, was later extended to radiative muon capture by Fearing (Fe 66), and it is in this latter form that the model is reviewed here.

C. Giant Dipole Resonance Model

The principal idea of the giant dipole resonance model is to relate the dipole part of the muon capture rate to an integral over the experimental photo-absorption cross section. To obtain the relation it is first assumed that isospin is a good quantum number. Thus for a nuclear ground

state $|a\rangle$ having total isospin T=0 (Fe 66) :

$$\sum_{\bar{a}b} \frac{k(k_m^{ab}-k)^2}{m_\mu^3} |\langle b|\sum_i \mathcal{J}_-(i) \exp(-i\vec{S}_{ab}\cdot\vec{r}_i)|a\rangle|^2$$

$$= \frac{1}{2}\sum_{\bar{a}b'} \frac{k(k_m^{ab}-k)^2}{m_\mu^3} |\langle b'|\sum_i J_z(i) \exp(-i\vec{S}_{ab}\cdot\vec{r}_i)|a\rangle|^2$$

$$= 2\pi \sum_{\bar{a}b'} \frac{k(k_m^{ab}-k)^2}{m_\mu^3} \sum_{lm} |\langle b'|\sum_i J_z(i) j_l(S_{ab}r_i) Y_l^m(\Omega r_i)|a\rangle|^2 \qquad 4\text{-}11$$

The sum on b' runs over excited T=1 states of the parent nucleus. The maximum photon energy in this expression is still k_m^{ab}, corresponding to the proper transition energy. Hence :

$$k_m^{ab} = E_m - E_{b'a} \qquad 4\text{-}12$$

with E_m as defined in Eq. 4-1.

In their examination of the dipole (l=1) part of the ordinary muon capture analogue of Eq. 4-11, Foldy and Walecka (Fo 64) noted that the dipole matrix element could be written as the product of the matrix element of an unretarded part $\frac{S_{ab}r_i}{3}$ and an elastic form factor F_{el}. With this prescription the dipole part of Eq. 4-11 becomes :

$$\mathcal{D} \equiv \frac{1}{6}\sum_{\bar{a}b'} \frac{k(k_m^{ab}-k)^2}{m_\mu^3} |F_{el}|^2 (S_{ab})^2 |\langle b'|\sum_i J_z(i)\vec{r}_i|a\rangle|^2 \qquad 4\text{-}13$$

The closure-harmonic oscillator model was used to compute the dipole term appearing in Eq. 4-11 and the approximation to it used in Eq. 4-13. Besides giving a check on the accuracy of the approximation, this calculation also yielded a check on the formulae used by Foldy and Walecka (Fo 64). The calculation shows :

$$2\pi \sum_{\bar{a}b'} \sum_{m} |\langle b'|\sum_i J_z(i) j_1(S_{ab} r_i) Y_1^m(\Omega r_i)|a\rangle|^2$$

$$= 10\eta^2\{1-\tfrac{1}{2}\eta^2+\tfrac{33}{200}\eta^4-\tfrac{11}{400}\eta^6+\tfrac{39}{11200}\eta^8+\dots\}\exp(-\eta^2/2) \qquad \text{4-14}$$

$$\tfrac{1}{6}\sum_{\bar{a}b'} |F_{el}|^2 (S_{ab})^2 |\langle b'|\sum_i J_z(i)\vec{r}_i|a\rangle|^2$$

$$= 10\eta^2\{1-\tfrac{1}{2}\eta^2+\tfrac{7}{80}\eta^4-\tfrac{1}{160}\eta^6+\tfrac{1}{3600}\eta^8\}\exp(-\eta^2/2) \qquad \text{4-15}$$

Here :

$$\eta^2 = (sb)^2 = b^2[k_m^2 - 2k(k_m-k)(1-y)]$$
$$y = \hat{k}\cdot\hat{\nu} \qquad \text{4-16}$$

and b is the oscillator parameter. For ^{40}Ca, taking b=2.03 fm and k_m=90 Mev, it is evident that Eq. 4-14 differs from Eq. 4-15 by at most a few percent for any value of the photon momentum k, so that Eq. 4-13 is indeed a good approximation to the dipole part of Eq. 4-11.

The nuclear matrix element which appears in Eq. 4-13 is precisely that which describes nuclear photo-absorption in the unretarded dipole approximation :

$$\sigma_\gamma(E) = \tfrac{1}{3}\pi^2\alpha \sum_{\bar{a}b'} (E_{b'}-E_a)|\langle b'|\sum_i J_z(i)\vec{r}_i|a\rangle|^2 \delta(E-E_{b'}-E_a) \qquad \text{4-17}$$

Using Eqs. 4-12 and 4-17, Eq. 4-13 becomes :

$$\mathcal{D} = \frac{1}{2\pi^2\alpha}\int_0^{E_m} dE\left\{\frac{\sigma_\gamma(E)}{E}\, k\,\frac{(E_m-E-k)^2}{m_\mu^3}\,\theta(E_m-E-k)\,|F_{el}(k_m^{ab}=E_m-E)|^2 \times[(E_m-E)^2-2k(E_m-E-k)(1-y)]\right\}$$

$$\simeq \frac{1}{2\pi^2\alpha}|F_{el}(E_{res})|^2\int_0^{E_m} dE\left\{\frac{\sigma_\gamma(E)}{E}\, k\,\frac{(E_m-E-k)^2}{m_\mu^3}\,\theta(E_m-E-k) \times[(E_m-E)^2-2k(E_m-E-k)(1-y)]\right\}$$

$$\mathcal{D} \equiv |F_{2L}(E_{res})|^2 \{Q_1(k) + Q_2(k)(1-y)\}$$

4-18

where $\theta(x)$ is the unit step function, E_{res} is the energy of the giant dipole resonance and the functions $Q_1(k)$ and $Q_2(k)$ are defined :

$$Q_1(k) \equiv \frac{k}{2\pi^2 \alpha m_\mu^3} \int_0^{E_m} dE \, \frac{\sigma_\gamma(E)}{E} \, \theta(E_m - E - k)(E_m - E - k)^2 (E_m - E)^2$$

$$Q_2(k) \equiv \frac{-k^2}{\pi^2 \alpha m_\mu^3} \int_0^{E_m} dE \, \frac{\sigma_\gamma(E)}{E} \, \theta(E_m - E - k)(E_m - E - k)^3$$

4-19

Hence the dipole part of the capture rate is related directly to the total photo-abscrption cross section, insofar as the squared nuclear matrix element of the form in Eq. 4-11 is concerned.

The latter qualification is necessary in that generally the nuclear matrix elements are functions of nucleon spins and momenta. The nucleon spins, however, are eliminated from consideration via the relations between squared nuclear matrix elements presented in Sec. III.C.1, so that the matrix elements appear either as in Eq. 4-11, or contain additional factors of nucleon momenta - the so-called "velocity terms". As the numerical results in the next chapter will show, the velocity terms play a lesser role quantitatively in radiative muon capture. In the giant dipole resonance model, the velocity terms and the other multipole terms (non-dipole contributions) were evaluated via the closure-harmonic oscillator model, using either

simple oscillator or SBL wave functions.

D. Modifications to the Usual Closure Approximation

In discussing, at the end of section IV.B, the failure of the closure-harmonic oscillator model to predict the correct dipole transition strength, the strong k_m dependence of the capture rate was mentioned. This dependence is of immediate concern because the average maximum photon energy k_m is not determined a priori by the model, although fixing k_m is highly desireable from the point of view of eliminating uncertainties due to the nuclear physics. A similar difficulty arises in a closure-harmonic oscillator model description of ordinary muon capture, where the parameter to be fixed is the average neutrino energy . A technique employing nuclear sum rules (Do 72, Be 73, Go 74, Ko 76) which involves modification of the simple closure approximation has been successful in largely eliminating the dependence of the ordinary capture rate. Recently the method has been extended to radiative capture by Sloboda and Fearing (Sl 78), whose work appears as Chapter VI here. A study of the conditions of applicability of the modified closure approach was undertaken by Rosenfelder (Ro 77), who demonstrated its suitability for the muon capture problem.

Ideally a combination of the giant dipole resonance and modified closure-harmonic oscillator models should result in

a model which is superior to either of the two. The realization of this combination meets with some difficulties in practice, however, as the simple sum rule arising in the modified closure-harmonic oscillator model does not emerge intact in attempts to modify the giant dipole resonance model. The proposed combined model would require a sum rule for the other multipoles which must be of higher order than the first order one described in Chapter VI. This is so because for the first order sum rule, the dipole part of the main nuclear matrix element squared for RMC yields the same sum as the full main nuclear matrix element squared itself. Higher order sum rules may be derived (Er 70) but their complexity makes them difficult to use.

The nuclear models outlined above complete our theoretical description of radiative muon capture. In the following chapter numerical results are presented.

V.

RESULTS

A. RMC Observables

By way of introduction to the numerical results, a summary of observables for radiative muon capture follows.

The probability for radiative muon capture from an initial nuclear state |a> to a final nuclear state |b> was given in Eq. 3-1 as :

$$P_{ab}(\vec{k},\vec{\nu},\lambda,\vec{s})\,dk\,d\Omega_k\,d\Omega_\nu = \frac{\alpha}{4(2\pi)^4}\,|\varphi_\mu|^2_{av}\sum_{\bar{a}b} k(k_m^{ab}-k)^2|M_{ba}|^2\,dk\,d\Omega_k\,d\Omega_\nu \qquad 3\text{-}1$$

Specializing to doubly-closed shell nuclei via the relations between squared nuclear matrix elements, Eqs. 3-11a and 3-11b, and using the closure approximation detailed in Eqs. 4-2 to 4-4, the total transition probability from the initial state |a> to all final states |b> is :

$$W(x,\vec{k},\vec{\nu},\lambda,\vec{s})\,dx\,d\Omega_k\,d\Omega_\nu = \sum_b P_{ab}(\vec{k},\vec{\nu},\lambda,\vec{s})\,dx\,d\Omega_k\,d\Omega_\nu$$
$$= \frac{\alpha}{2(2\pi)^4 m_\mu^2}\,|\varphi_\mu|^2_{av}k_m^4\,x(1-x)^2\mathcal{M}^2\,dx\,d\Omega_k\,d\Omega_\nu \qquad 5\text{-}1$$

where $x=k/k_m$. The squared matrix element $\mathcal{M}^2$ is a lengthy expression involving the weak coupling constants, $\vec{\nu}$, $\vec{k}$, x, λ, and products of nuclear matrix elements of the form described by Eq. 4-5. $\mathcal{M}^2$ is presented explicitly in Appendix C.

As enumerated by Rood and Tolhoek (Ro 65), the most important observables are :

1. the photon spectrum

$$N(x)\,dx = \sum_{\lambda} \iint d\Omega_k\, d\Omega_\nu\; W(x,\vec{k},\vec{\nu},\lambda,\vec{\xi})\,dx \qquad 5\text{-}2$$

2. the photon-neutrino directional correlation

$$W(x,\theta_{\nu\gamma}) = \sum_{\lambda} \int \frac{d\Omega_\nu}{4\pi}\; W(x,\vec{k},\vec{\nu},\lambda,\vec{\xi}) \qquad 5\text{-}3$$

3. the circular polarization of the emitted photons

$$P_\gamma(x) = \frac{\iint d\Omega_k\, d\Omega_\nu \left[W(x,\vec{k},\vec{\nu},\lambda=+1,\vec{\xi}) - W(x,\vec{k},\vec{\nu},\lambda=-1,\vec{\xi})\right]}{\iint d\Omega_k\, d\Omega_\nu \left[W(x,\vec{k},\vec{\nu},\lambda=+1,\vec{\xi}) + W(x,\vec{k},\vec{\nu},\lambda=-1,\vec{\xi})\right]} \qquad 5\text{-}4$$

4. the photon-muon polarization directional correlation

$$W(x,\theta_{\mu\gamma}) = \sum_{\lambda} \int d\Omega_\nu\; W(x,\vec{k},\vec{\nu},\lambda,\vec{\xi}) \qquad 5\text{-}5$$

In practice the measurement of the photon-neutrino directional correlation is the most challenging. Conceivably for light nuclei the photon and the nuclear recoil could be observed in coincidence, although the possibility of neutron emission from the nucleus would complicate matters considerably. At present the photon spectrum and the photon-muon polarization correlation have been investigated experimentally, so that our attention is focussed on the computation of these quantities.

B. Photon Spectrum

The differential photon spectrum has previously been calculated by Rood and Tolhoek (Ro 65) using the closure-harmonic oscillator model and by Fearing (Fe 66) using the giant dipole resonance model. These authors performed Pauli reductions of essentially the same relativistic Hamiltonian used in this work, keeping terms in the effective Hamiltonian through $O(1/m)$. Here all terms through $O(1/m^2)$ arising from a Foldy-Wouthuysen reduction of the relativistic Hamiltonian of Eq. 2-1 were retained, resulting in the effective Hamiltonian which appears in Appendix B. The effective Hamiltonian was squared as described in Chapter III, and numerical work was done using the FORTRAN program NEWRMC, which evolved from a program written by H.W. Fearing. Our photon spectra were calculated using a squared matrix element complete through $O(1/m^2)$. In order to facilitate the comparison of results, the following set of parameters has been defined for ^{40}Ca :

$b=2.03$ fm.

$\nu=85$ Mev.

$k_m=87.6$ Mev.$= 1.03 \times \nu$

$g_S=0.0$

$g_T=0.0$

$g_M=3.7$

$g_V=1.0$

$g_A=-1.25$

$g_P=-8.75= 7 \times g_A$

This set will be referred to as the standard parameter set since it is composed mainly of parameters which have been used most often in previous RMC calculations (Ro 65, Fe 66, Ro 74). It should be noted that the parameters in the standard set are not necessarily the best ones. Choices for all weak couplings except g_P do however reflect the current state of experimental knowledge of these quantities (Kr 75). For g_P, the Goldberger-Treiman value $g_P=7xg_A$ was used. The oscillator parameter b was chosen to reproduce the experimental RMS charge radius for ^{40}Ca (Lu 63). The choices for the average neutrino energy ν and the average maximum photon energy k_m were somewhat arbitrary although they tend to follow historical precedent (Ro 65, Fe 66). The ratio $k_m/\nu=1.03$ is slightly larger than the Rood and Tolhoek estimate of 1.02 obtained for ^{16}O by equating results of a closure calculation with those obtained by summing partial transitions, and smaller than our own estimate for ^{40}Ca of 1.06, based on somewhat different considerations explained in Chapter VI. In instances where a relative rate (ratio of the rate for radiative to ordinary capture) is presented, the ordinary capture rate was calculated using the effective Hamiltonian of Fujii and Primakoff (Fu 59, Pr 59). Terms of $O(1/m^2)$ in the ordinary rate were not included since these have been shown to contribute <5% (Fr 66, Oh 66).

Figure 2 shows our $O(1/m^2)$ photon spectrum and a corresponding spectrum derived using the squared matrix

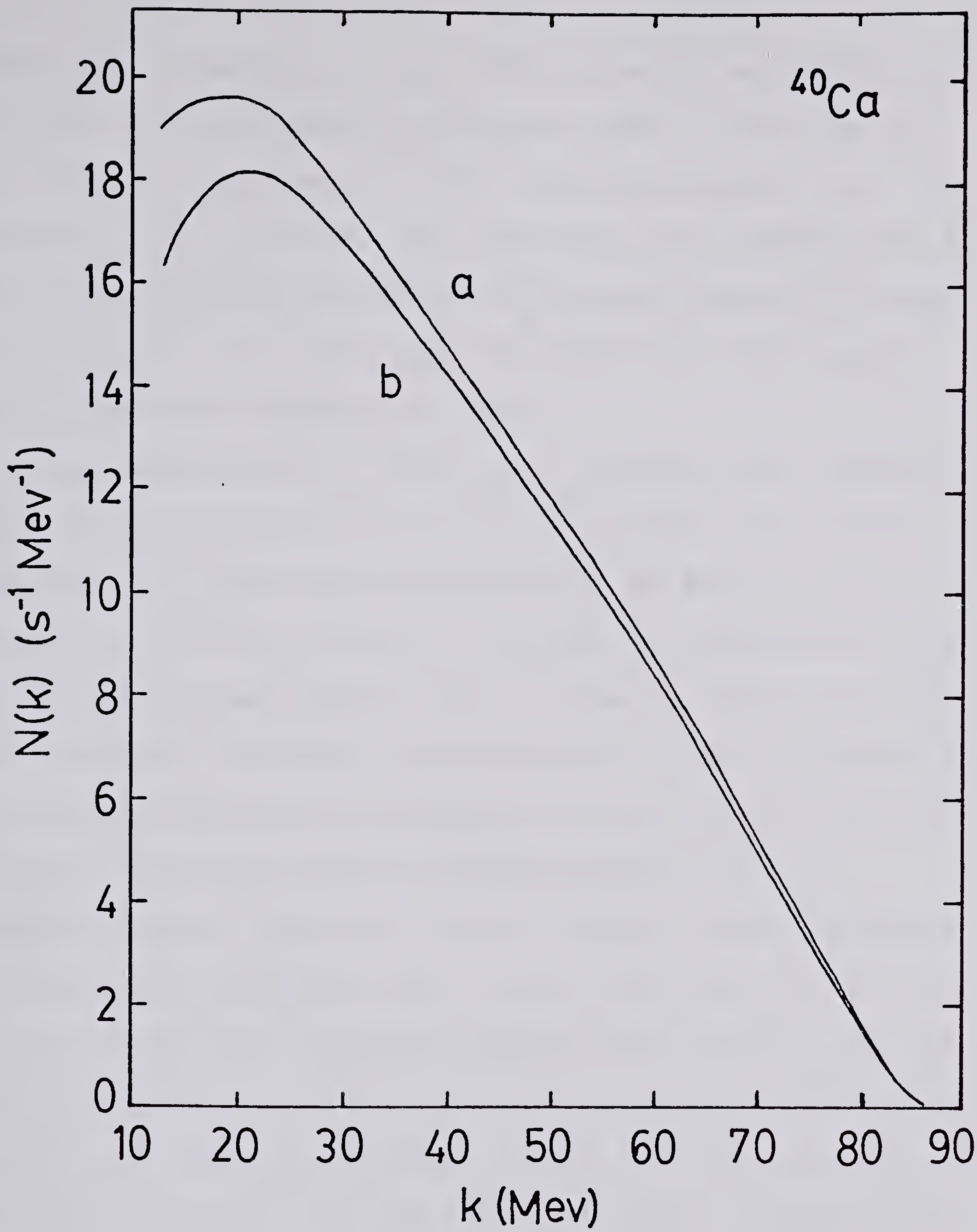

Fig. 2. Contribution of previously neglected $O(1/m^2)$ terms in the squared matrix element to the photon spectrum for ^{40}Ca, calculated in the CHO model
a - our $C(1/m^2)$ result
b - using the squared matrix element from Fearing (Fe 66).

element from Fearing (Fe 66), both of which were calculated with the closure-harmonic oscillator model. Note that for k>30 Mev the additional $O(1/m^2)$ terms contribute only a few percent to the spectrum. This indicates good convergence of the non-relativistic expansion of the Hamiltonian in powers of $(1/m)$, as it has also been confirmed that the higher order terms are individually small.

The contribution to the photon spectrum from various parts of the squared matrix element has been examined in some detail by Rood and collaborators (Ro 65,Ro 75) in efforts to deal only with the numerically significant terms. The intent of the present study has been to retain all terms where this was possible; nevertheless it remains instructive to identify specific contributions to the photon spectrum. Figure 3 shows the photon spectrum computed in the closure-harmonic oscillator model keeping terms (including velocity terms) through $O(1)$, $O(1/m)$, and $O(1/m^2)$, and that portion of the rate through $O(1/m^2)$ coming solely from the velocity terms, ie. those containing an explicit $\vec{P}$. The most numerically significant $O(1/m^2)$ terms come from the square of $O(1/m)$ terms in the effective Hamiltonian. These larger $O(1/m^2)$ terms have been included previously (Ro 65). In figure 4 the contribution of the muon radiating diagram and its interference with all other diagrams (fig. 1 b-e) is depicted. As has been known for some time, the contribution to the photon spectrum from the muon radiating diagram is a major one (Pr 59). Figure 5 shows the influence of the

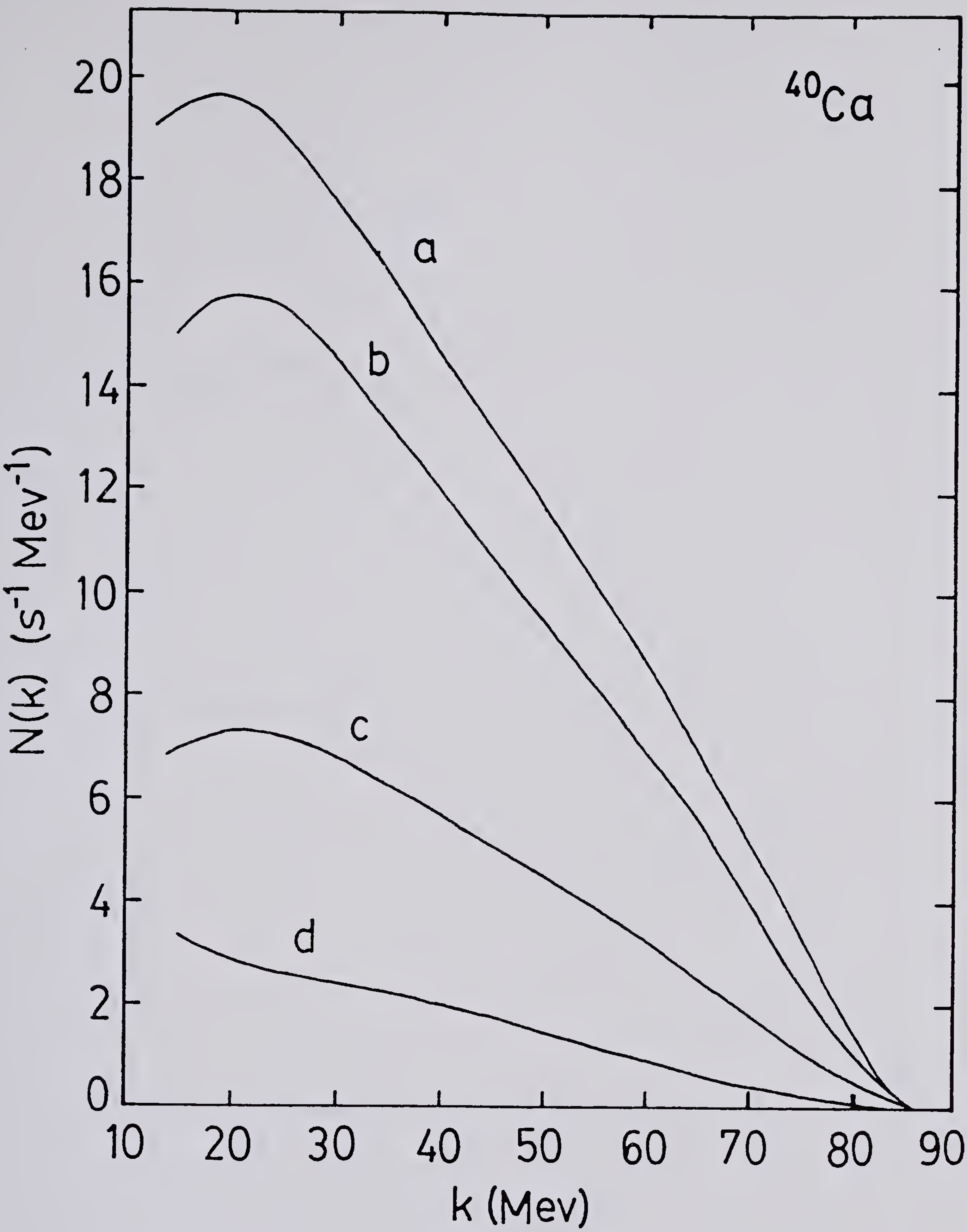

Fig. 3. Decomposition of the photon spectrum for ^{40}Ca according to powers of (1/m) appearing in the squared matrix element, and the contribution of the velocity terms, calculated in the CHO model

a - through O($1/m^2$)

b - through O($1/m$)

c - through O(1)

d - velocity terms through O($1/m^2$).

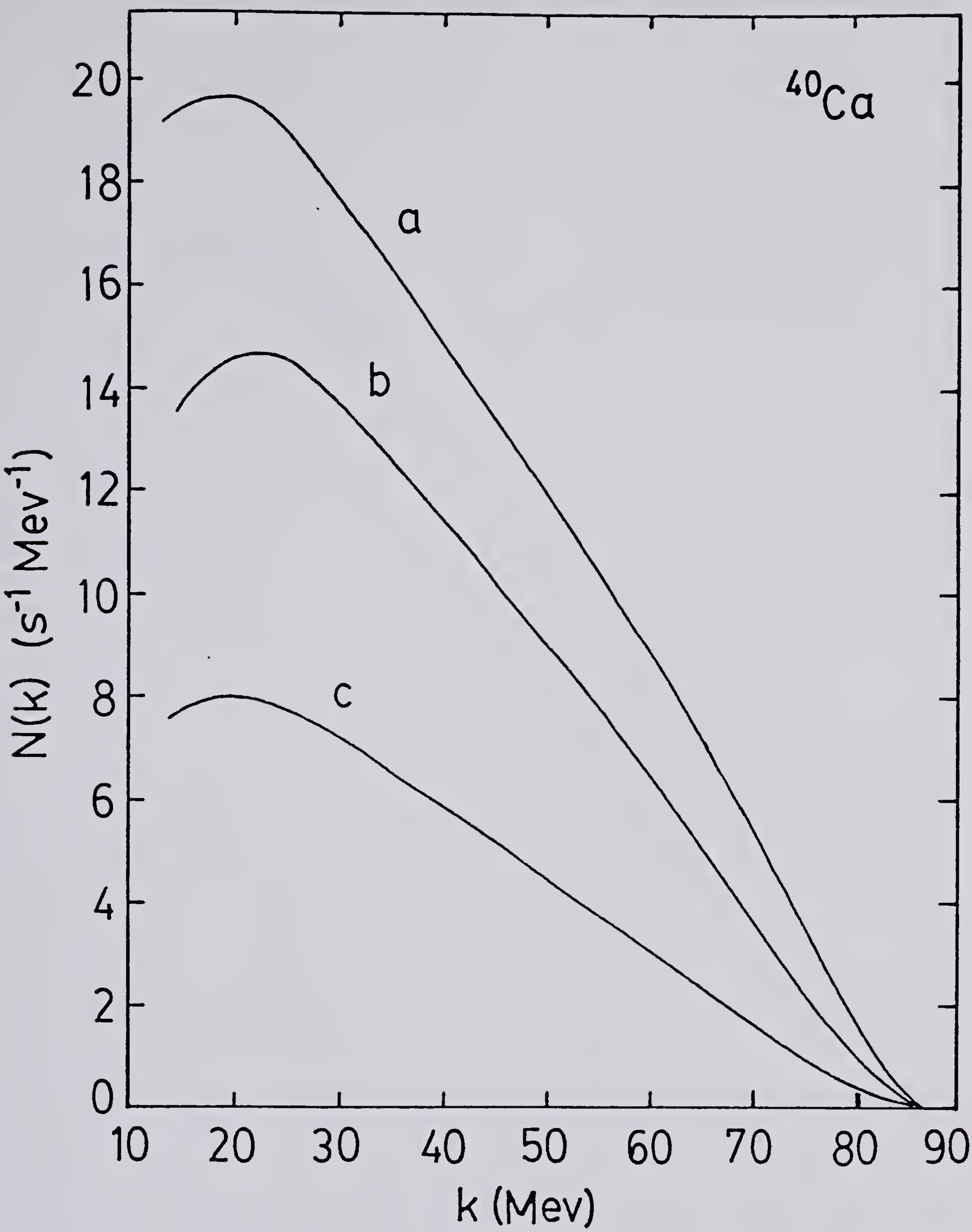

Fig. 4. Contribution of the muon radiating diagram and its interference with all other standard theory diagrams to the photon spectrum for ^{40}Ca, calculated in the CHO model
a - complete $O(1/m^2)$ result
b - muon radiating diagram plus interferences
c - muon radiating diagram alone.

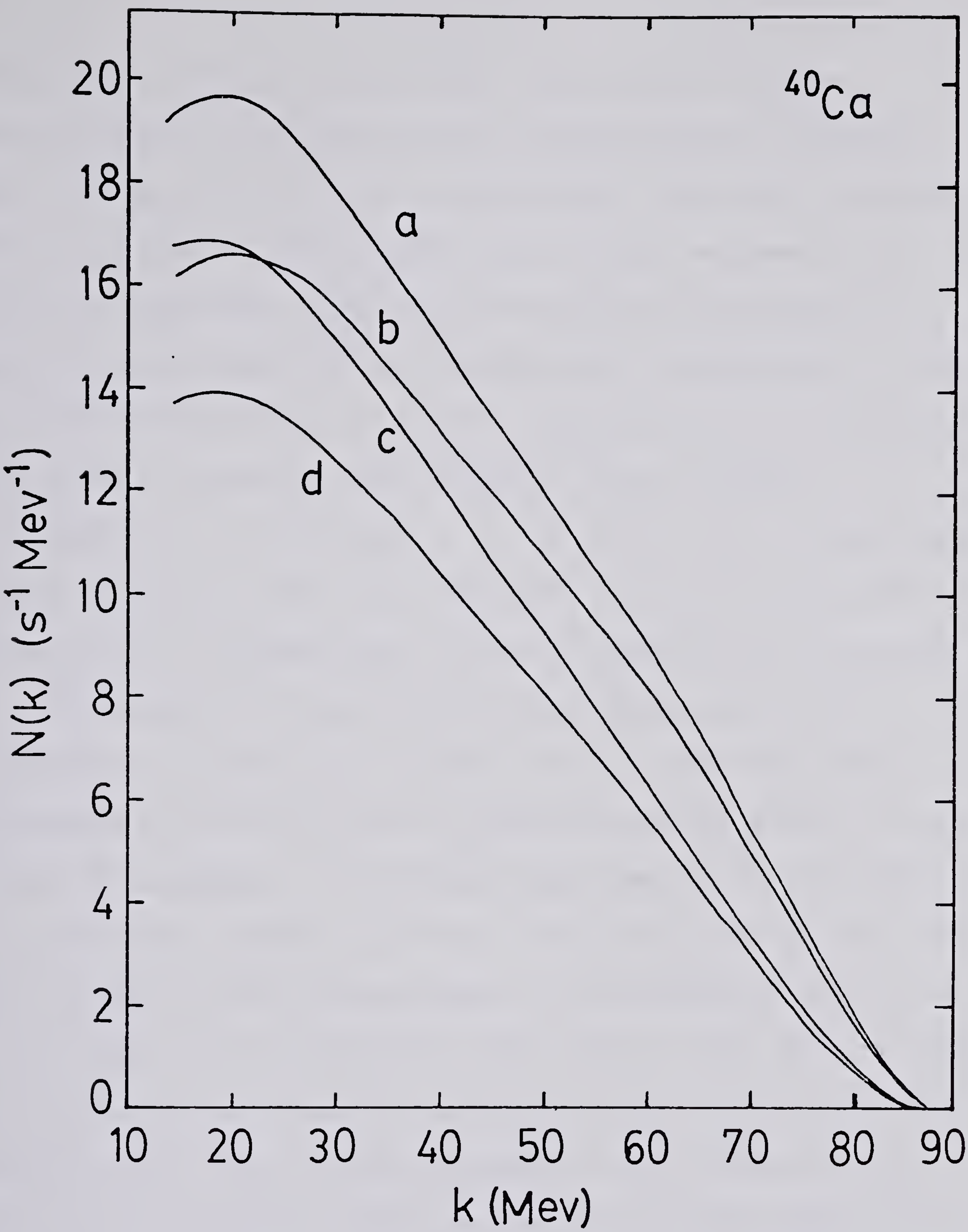

Fig. 5. Contribution to the ^{40}Ca photon spectrum from the weak magnetism and induced pseudoscalar couplings
a - complete $O(1/m^2)$ result, standard couplings
b - $g_M=0$
c - $g_P=0$
d - $g_M=g_P=0$

couplings g_P and g_M. Notice that the g_P terms become relatively more important as the photon energy increases while the g_M terms do not. The diagrams where the anomalous magnetic moment of the proton and neutron represent the source of radiation give a net contribution depicted in figure 6. For these latter diagrams the contribution to the rate is small and they are often neglected. It is estimated from figure 5 however that dropping these diagrams corresponds to a variation in g_P of about $\pm\ 2g_A$ at any given photon energy k. From this viewpoint the small contributions are seen to be significant and hence should not be dropped.

The advent of recent total photoabsorption cross sections for medium mass nuclei (Ah 75) warranted fresh computations using the giant dipole resonance model. In the course of examining the existing application of the model to radiative muon capture (Fe 66), two major improvements were made. First, it was noticed that an inconsistent choice of the average neutrino energy in the latter study had resulted in a giant dipole resonance model estimate for the relative spectrum which was too low. Allegedly the values ν=85 Mev and k_m=88 Mev had been used in closure-harmonic oscillator and giant dipole resonance model calculations of the relative photon spectrum, however the giant dipole resonance model calculation of the ordinary rate was not in keeping with the above value for ν. The main nuclear matrix element squared in the giant dipole resonance model appears as :

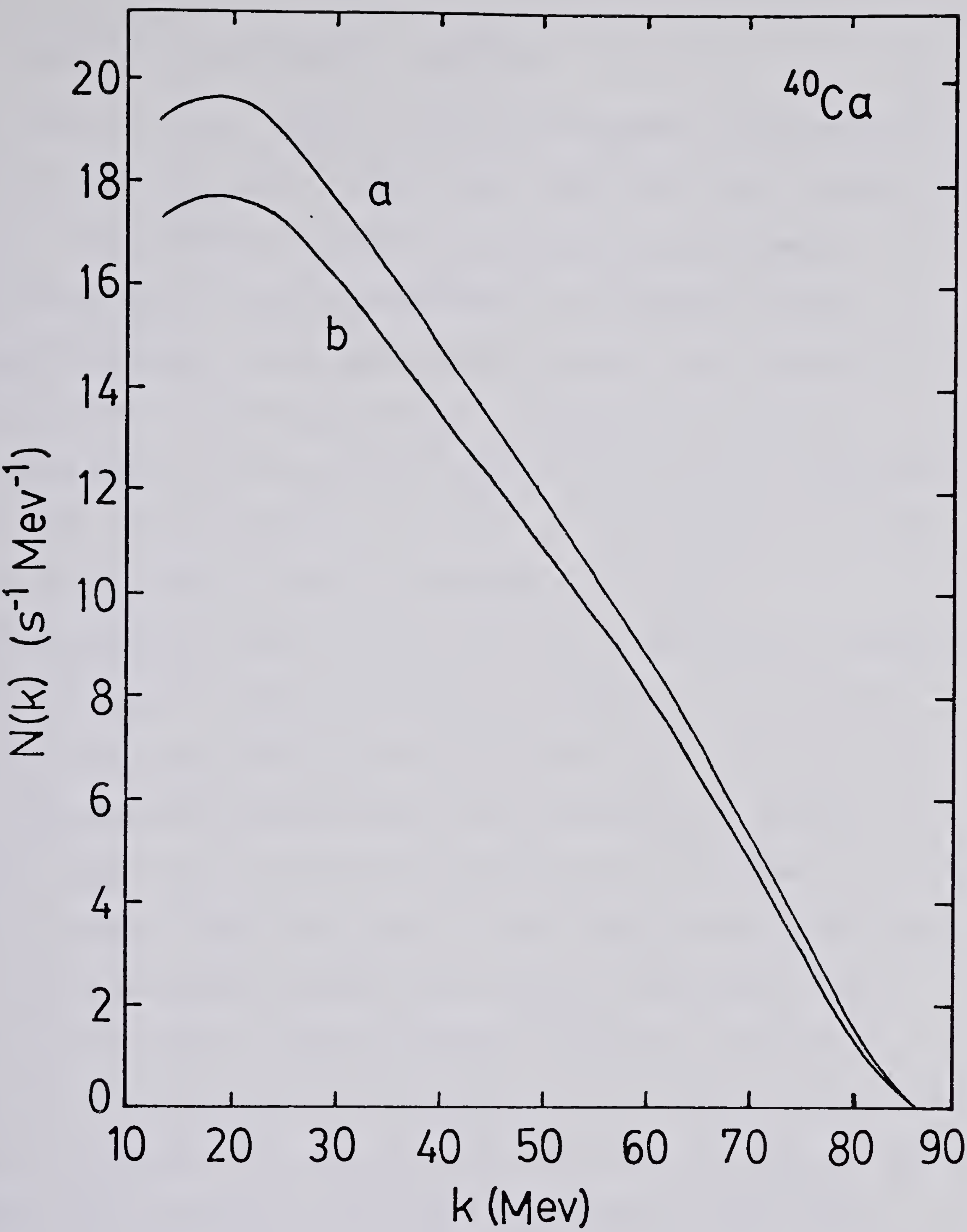

Fig. 6. Effect of neglecting the anomalous magnetic moment of the proton and neutron on the photon spectrum for ^{40}Ca, calculated in the CHO model
a - complete $O(1/m^2)$ result
b - $\chi_P = \chi_N = 0$.

$$\langle 1\rangle\langle 1\rangle^{*} = |F_{\ell\ell}|^2 (M_V^2)_{UD} + (M_V^2)_{OM} \qquad 5\text{-}6$$

The value 1.67 for $(M_V^2)_{OM}$ was taken directly from Foldy and Walecka (Fo 64), who in fact used ν=89.8 Mev. As a result $(M_V^2)_{OM}$ and the ordinary capture rate in the giant dipole resonance model were overestimated as compared to the closure-harmonic oscillator result, hence the relative capture rate in the giant dipole resonance model was underestimated. For ν=85 Mev $(M_V^2)_{OM}$=1.26, giving a value for the full matrix element, Eq. 5-6, of 2.28 compared to the value 2.70 used in (Fe 66). Secondly, the new photoabsorption data for ^{40}Ca (Ah 75) permitted the removal of an assumption made by Fearing concerning the relation between the then unmeasured photoproton cross section and the experimental photoneutron cross section of Baglin and Spicer (Ba 64). It had been assumed that the shapes of these cross sections were the same so that the photoneutron cross section was scaled to give the correct integrated cross section for proton photoproduction. Thus the relatively small low energy part of the (γ,p) cross section below the (γ,n) threshold was neglected. Our calculations using the Ahrens data, however, show that this part of the cross section is significant in the giant dipole resonance model because the low energy part of the photoabsorption cross section is the most strongly weighted in the photoabsorption integrals Q_1 and Q_2 of Eq. 4-19.

In figures 7 and 8 are shown the results of making the two aforementioned improvements. Figure 7 gives the

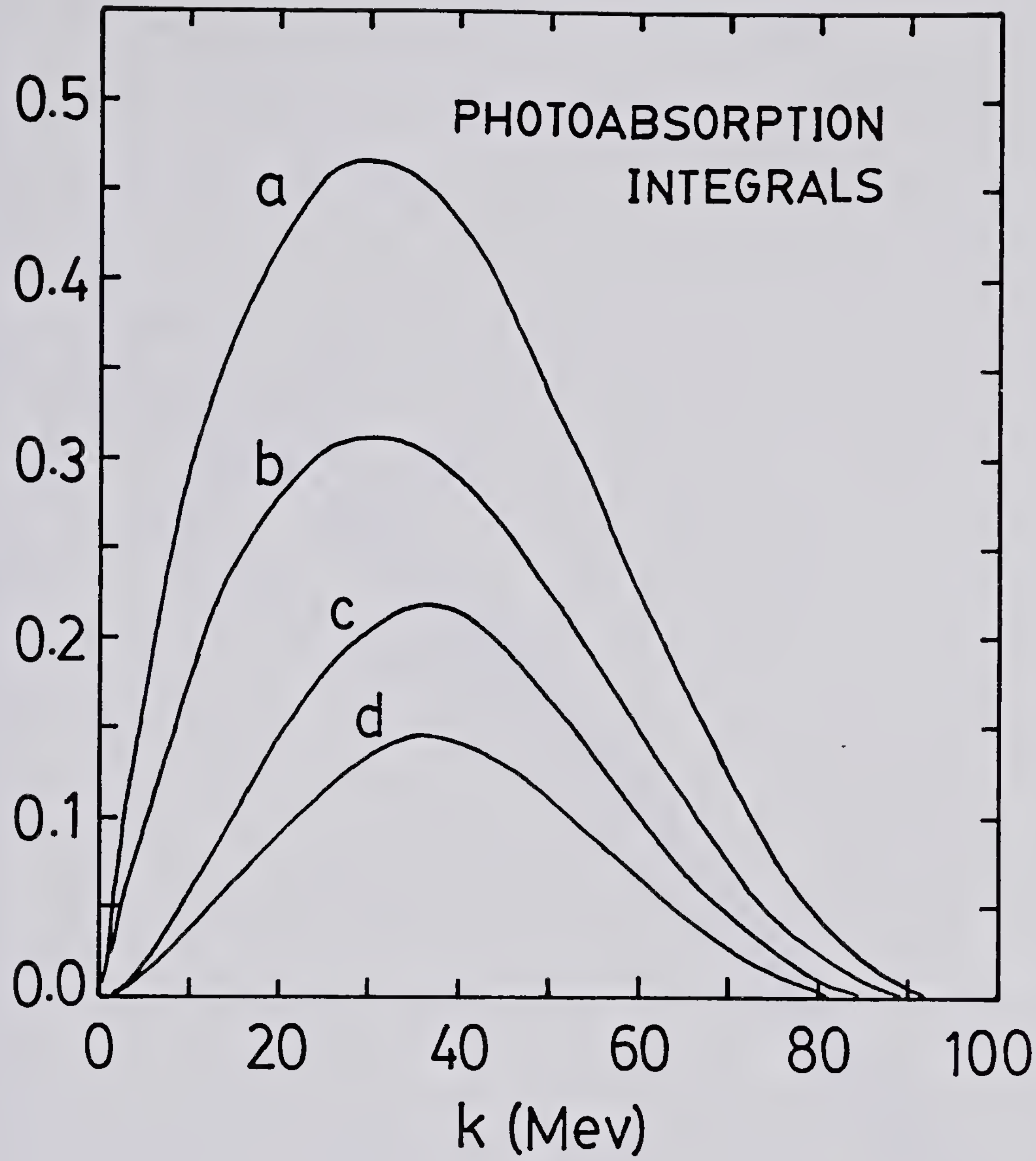

Fig. 7. Integrals Q_1 and Q_2 over the experimental photoabsorption cross section for ^{40}Ca
a - Q_1, data of Ahrens et al (Ah 75)
b - Q_1, Baglin and Spicer data (Ba 64)
c - Q_2, data of Ahrens et al (Ah 75)
d - Q_2, Baglin and Spicer data (Ba 64).

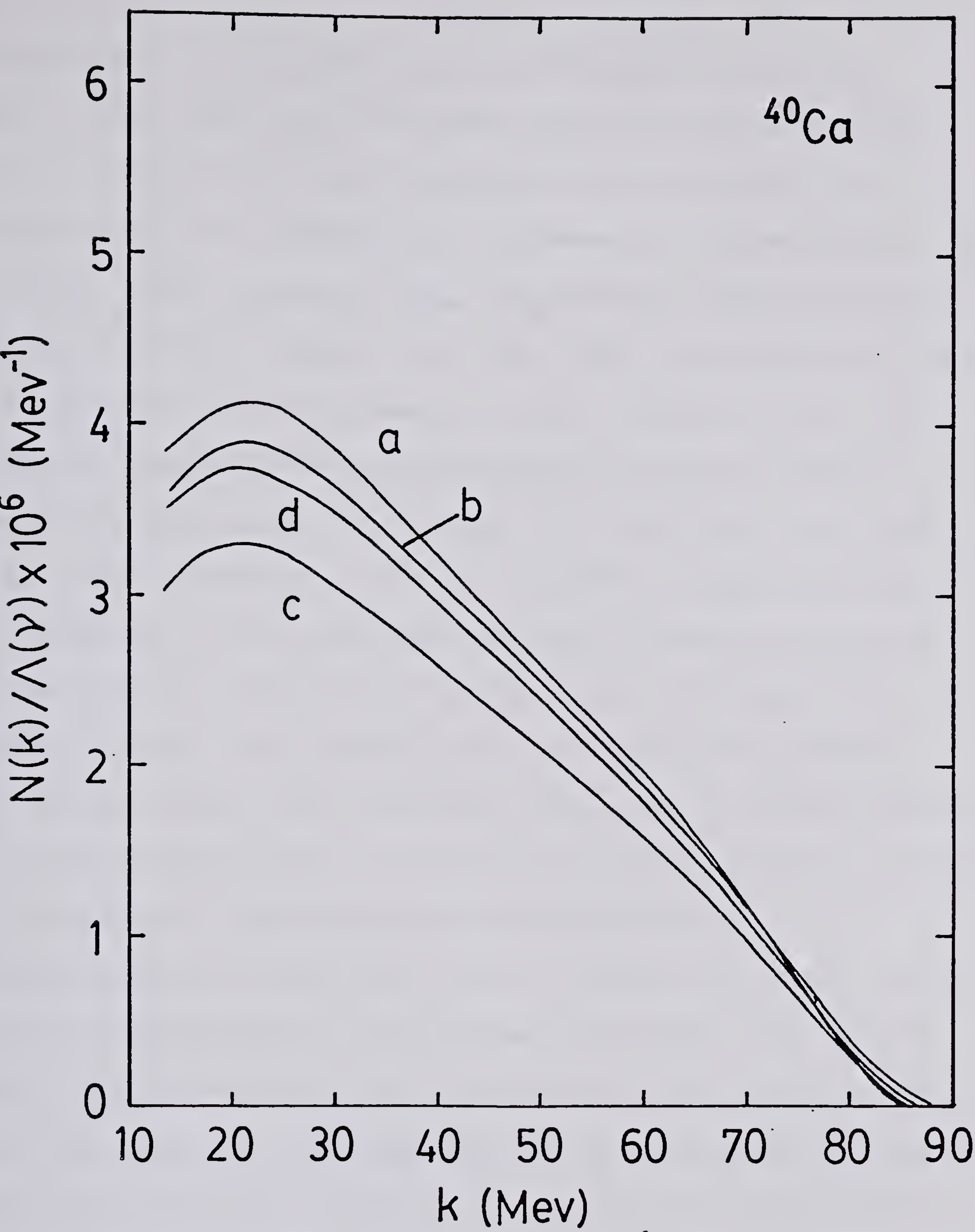

Fig. 8. The relative photon spectrum for ^{40}Ca, calculated in the GDR model using the squared matrix element from Fearing (Fe 66). The ordinary capture rate Λ is also given

a - CHO model result; $\Lambda = 4.36 \times 10^6$/s

b - using the photoabsorption data of Ahrens et al (Ah 75); $\Lambda = 3.58 \times 10^6$/s

c - Fearing's result using the Baglin and Spicer photoabsorption data (Ba 64) and the average energies $\nu = 89.8$ Mev, $k_m = 88$ Mev; $\Lambda = 2.98 \times 10^6$/s

d - using the Baglin and Spicer photoabsorption data; $\Lambda = 3.44 \times 10^6$/s.

integrals Q_1 and Q_2 over the scaled Baglin and Spicer (Ba 64) data and over the Ahrens (Ah 75) photoabsorption cross sections for ^{40}Ca. The Ahrens data is seen to be responsible for a substantial increase in the magnitudes of Q_1 and Q_2 . The quantity $(M_V^2)_{UD}$ appearing in the expression for the ordinary capture rate (Eq. 5-6) also increased from 2.56 (Baglin and Spicer data) to 3.84 (Ahrens data). Our relative photon spectrum computed in the giant dipole resonance model appears in figure 8, along with the curve calculated by Fearing (Fe 66). In order to best illustrate the effects of the improvements, our calculation used the matrix element derived by Fearing. Other multipole and velocity terms were calculated in the closure-harmonic oscillator model. The important difference which emerges is that the 20% reduction in the relative rate found by Fearing for the giant dipole model as compared to the closure-harmonic oscillator model becomes very small in the present calculation of these same quantities. For $60<k<80$ Mev, approximately 40% of this change in the relative rate is attributable to the emergence of better photoabsorption data, the remaining 60% being due to our more consistent calculation of $(M_V^2)_{OM}$, leading to a smaller result for the ordinary capture rate. Note however that since different choices of k_m and ν can cause variations in the relative rate which are different for different models, the striking agreement between the relative rates calculated in the CHO and GDR models for the standard values k_m=87.6 Mev, ν=85 Mev

must be regarded as something of a coincidence. Thus choosing k_m=84 Mev, ν=79 Mev (see figs. 18 and 19) one finds that the relative rates in the CHO and GDR models differ substantially for k>60 Mev. The reason is that while the high energy part of the GDR spectrum is fairly insensitive to the choice of k_m and ν, the CHO spectrum is cut off at $k=k_m$ by the closure approximation.

The main advantage in using the giant dipole resonance model to describe radiative muon capture is that the fraction of the rate due to dipole transitions (about 55% for ^{40}Ca) is directly evaluated using photoabsorption data. The importance of correctly evaluating the dipole contribution is illustrated in figure 9. The dipole part in the giant dipole resonance model amounts to only 66% of the dipole part in the closure-harmonic oscillator model, so that the absolute photon spectrum is substantially reduced in the giant dipole resonance theory, resulting in much better agreement with the experimentally observed absolute photon spectrum for ^{40}Ca.

The squared matrix element given in Appendix C depends on a large number of variables. It will prove instructive to examine the sensitivity of the photon spectrum to variations in these variables, always with an eye to the feasibility of obtaining information on the weak couplings. Variations were carried out with respect to the standard parameters.

Efforts to detect the presence of second class (G-parity violating) currents in nuclear beta decay have not

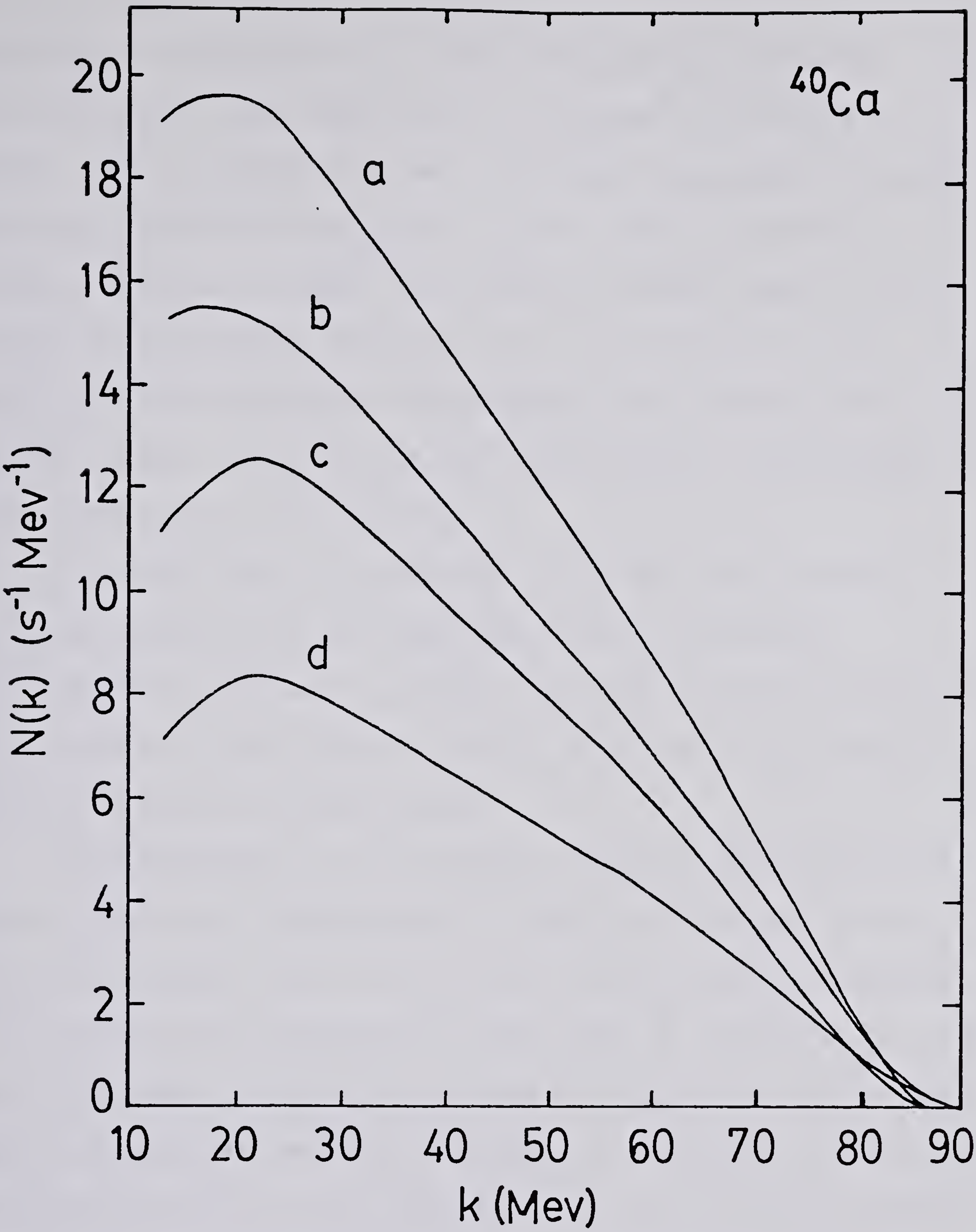

Fig. 9. Dipole contribution to the photon spectrum for ^{40}Ca
a - complete spectrum, CHO model
b - complete spectrum, GDR model
c - dipole part, CHO model
d - dipole part, GDR model.

succeeded in demonstrating that second class currents actually exist (for details, see reviews Te 77, Wu 77). Should they be found however, the usual assignment of weak coupling constants with $g_S=0.$, $g_T=0.$, would have to be altered. Figure 10 shows the relative photon spectrum for values of the scalar coupling $g_S=\pm1$. In figure 11 the relative photon spectrum is presented for values of the induced tensor coupling $g_T=\pm g_M$, with $g_M=3.7$, the standard value predicted by CVC theory.

As first noted by Wolfenstein (Wo 58), the photon spectrum exhibits strong dependence on the induced pseudoscalar coupling g_P. Figure 12 indicates the degree of sensitivity of the relative photon spectrum to g_P, and is included here for completeness.

The importance of the nuclear model in radiative muon capture has been investigated by Rood and Tolhoek (Ro 65). They established that the relative photon spectrum depends only slightly on the nuclear model used to calculate it. The basis for their conclusion involved comparisons for several approximations and models, including a statistical model and a closure-harmonic oscillator model. In the case of harmonic oscillator wave functions which are parametrized by the oscillator parameter b, the relative photon spectrum calculated here was found to be insensitive to modest variations in the value of b. This is illustrated in figure 13.

The Rood and Tolhoek study (Ro 65), while it utilized a

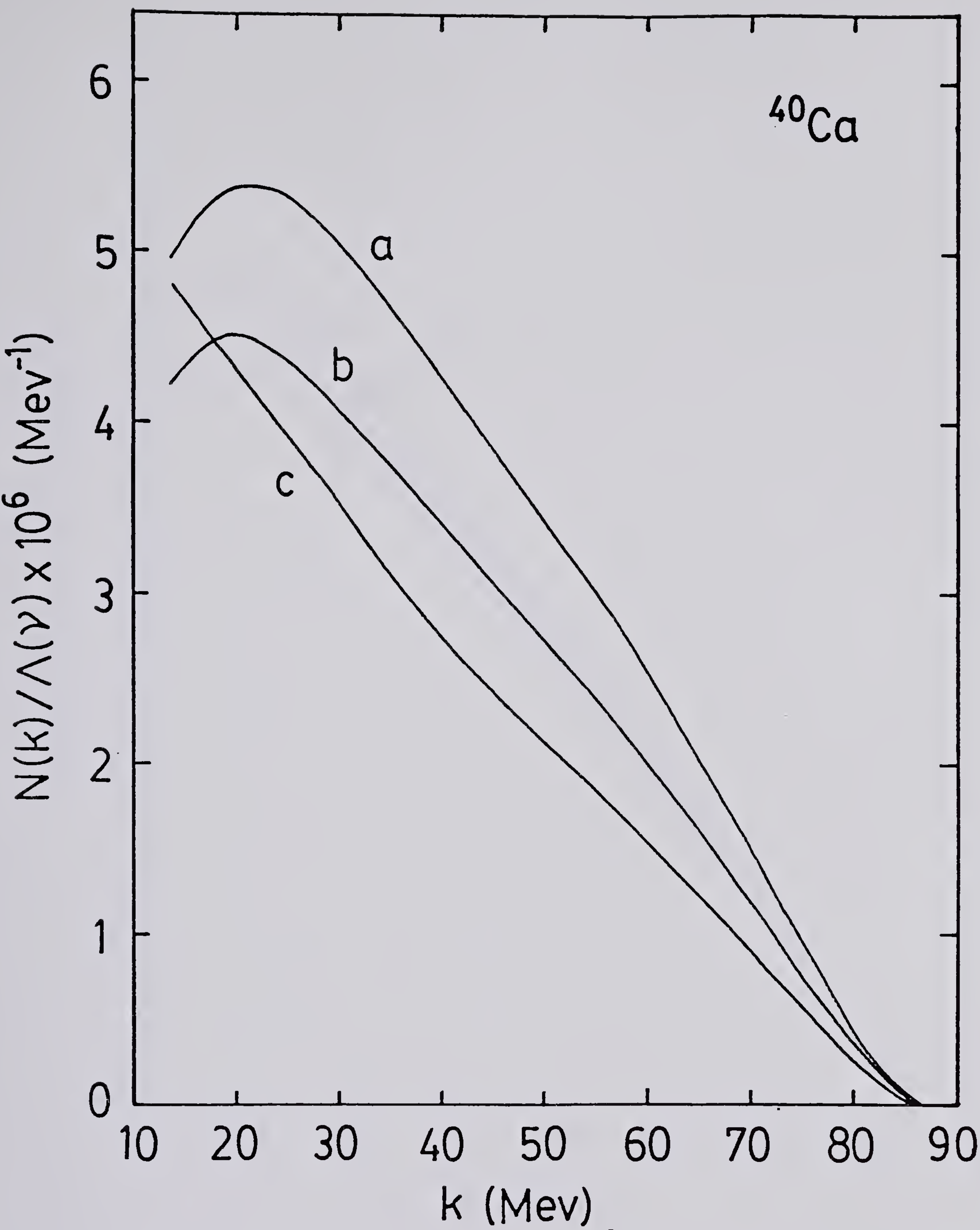

Fig. 10. Effect of varying g_s on the relative photon spectrum for ^{40}Ca, calculated in the CHO model. The ordinary capture rate Λ is also given.
a - g_s=-1. ; Λ =3.48x10^6/s
b - g_s= 0. ; Λ =4.36x10^6/s
c - g_s=+1. ; Λ =6.57x10^6/s.

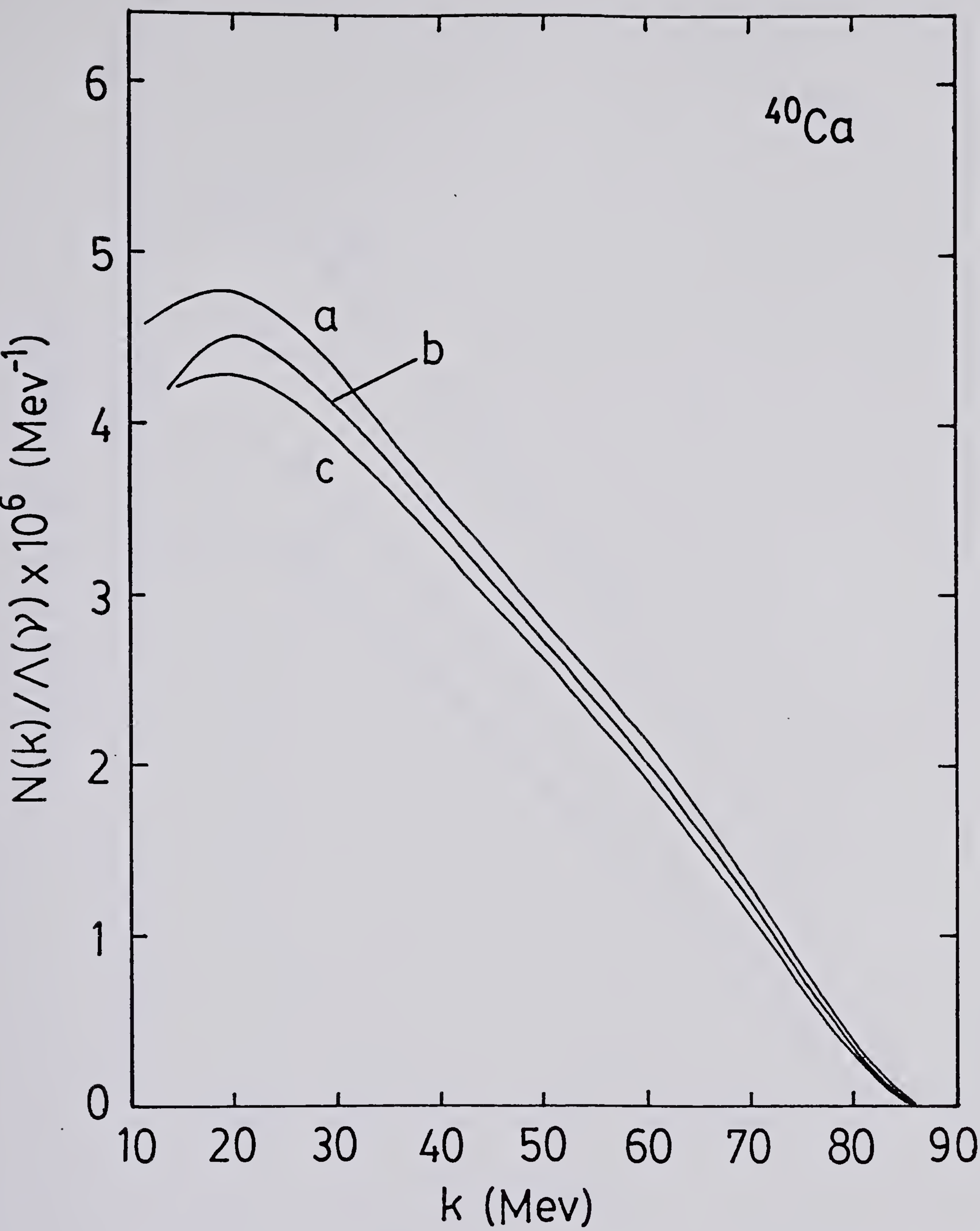

Fig. 11. Effect of varying g_T on the relative photon spectrum for ^{40}Ca, calculated in the CHO model. The ordinary capture rate Λ is also given.

a - $g_T=+g_M=+3.7$; $\Lambda=4.16\times10^6$/s

b - $g_T=0.$; $\Lambda=4.36\times10^6$/s

c - $g_T=-g_M=-3.7$; $\Lambda=4.60\times10^6$/s.

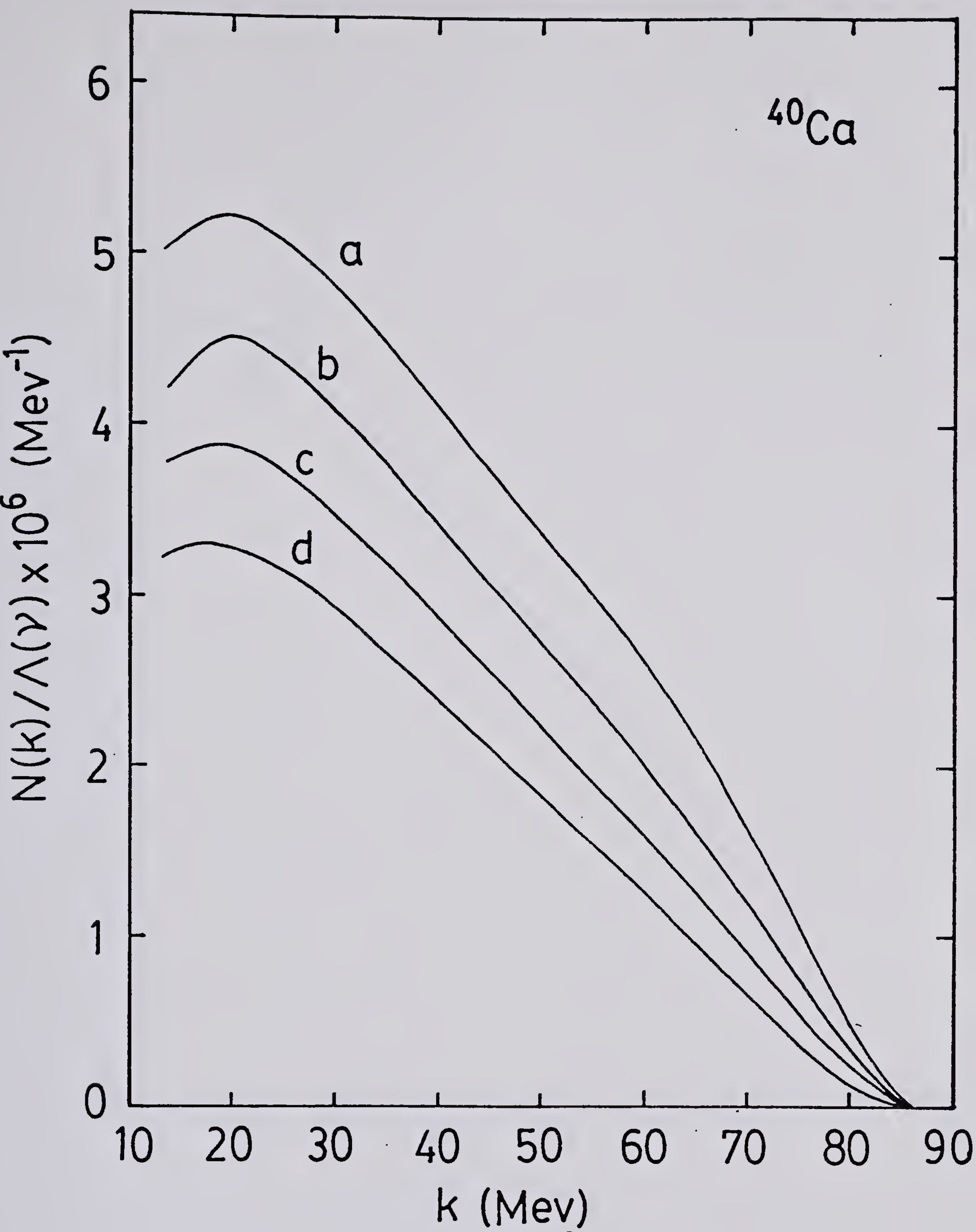

Fig. 12. Effect of varying g_P on the relative photon spectrum for ^{40}Ca, calculated in the CHO model. The ordinary capture rate is also given.

a - $g_P = 10 \times g_A = -12.5$; $\Lambda = 4.13 \times 10^6$/s
b - $g_P = 7 \times g_A = -8.75$; $\Lambda = 4.36 \times 10^6$/s
c - $g_P = 4 \times g_A = -5.0$; $\Lambda = 4.65 \times 10^6$/s
d - $g_P = 0.0$; $\Lambda = 5.11 \times 10^6$/s.

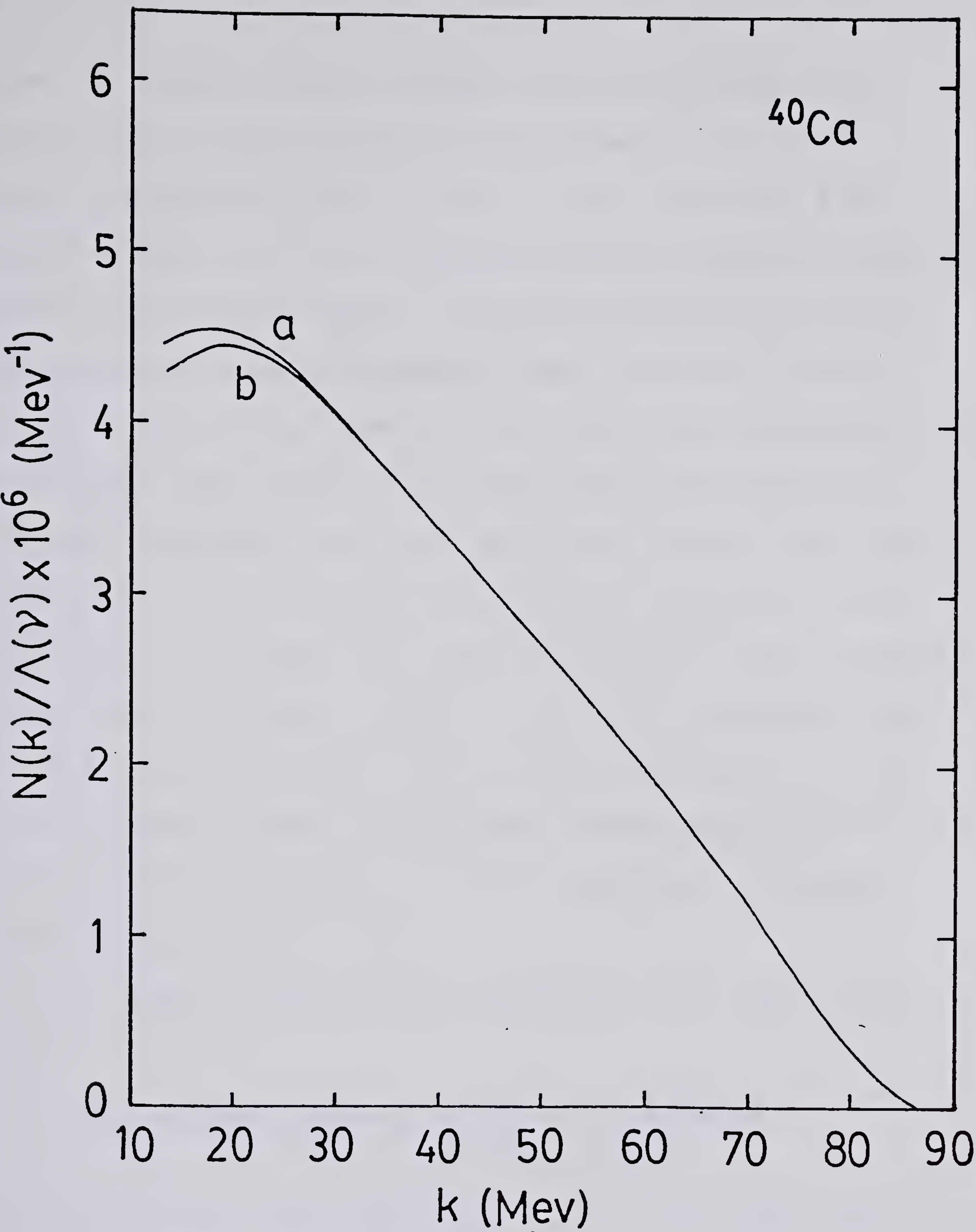

Fig. 13. Effect of varying the oscillator parameter b on the relative photon spectrum for ^{40}Ca, calculated in the CHO model. The ordinary capture rate Λ is also given.

a - b=1.90 fm ; $\Lambda = 4.01 \times 10^6$/s

b - b=2.15 fm ; $\Lambda = 4.68 \times 10^6$/s.

number of simple nuclear models, did not consider more sophisticated wave functions than those of the simple harmonic oscillator shell model. In fact the Rood and Tolhoek survey of nuclear models indicates that at least for doubly-closed shell nuclei, different elementary nuclear models predict relative capture rates which are in good agreement with one another. In our work the investigation of the nuclear wave function is taken one step further by computing the photon spectrum using the Hartree-Fock wave function for ^{40}Ca of Shao et al (Sh 73). As mentioned in Chapter IV, the results for the main nuclear matrix element squared given in Eqs. 4-6 and 4-7 are to be compared using slightly different values for b. This discrepancy is not crucial however since, as has been demonstrated earlier, the relative photon spectrum is quite insensitive to modest variations of b.

As a test of the SBL wave function, the charge form factor :

$$F_{ch}(q) = F(q) \times F_N(q) = \frac{1}{A} \frac{\langle a| \sum_{i=1}^{A} exp(i\vec{q}\cdot\vec{r}_i)|a\rangle}{(1+0.055q^2)^2} \qquad 5\text{-}7$$

was computed for ^{40}Ca. Here F(q) is the body form factor, $F_N(q)$ is the nucleon charge form factor, and $\vec{q}$ is the momentum transfer. The nucleon charge form factor was parametrized as in Lim (Li 73). Results for the SBL and simple harmonic oscillator wave functions, along with an experimental determination of the form factor by Bellicard et al (Be 67) are presented in figure 14. The closed form

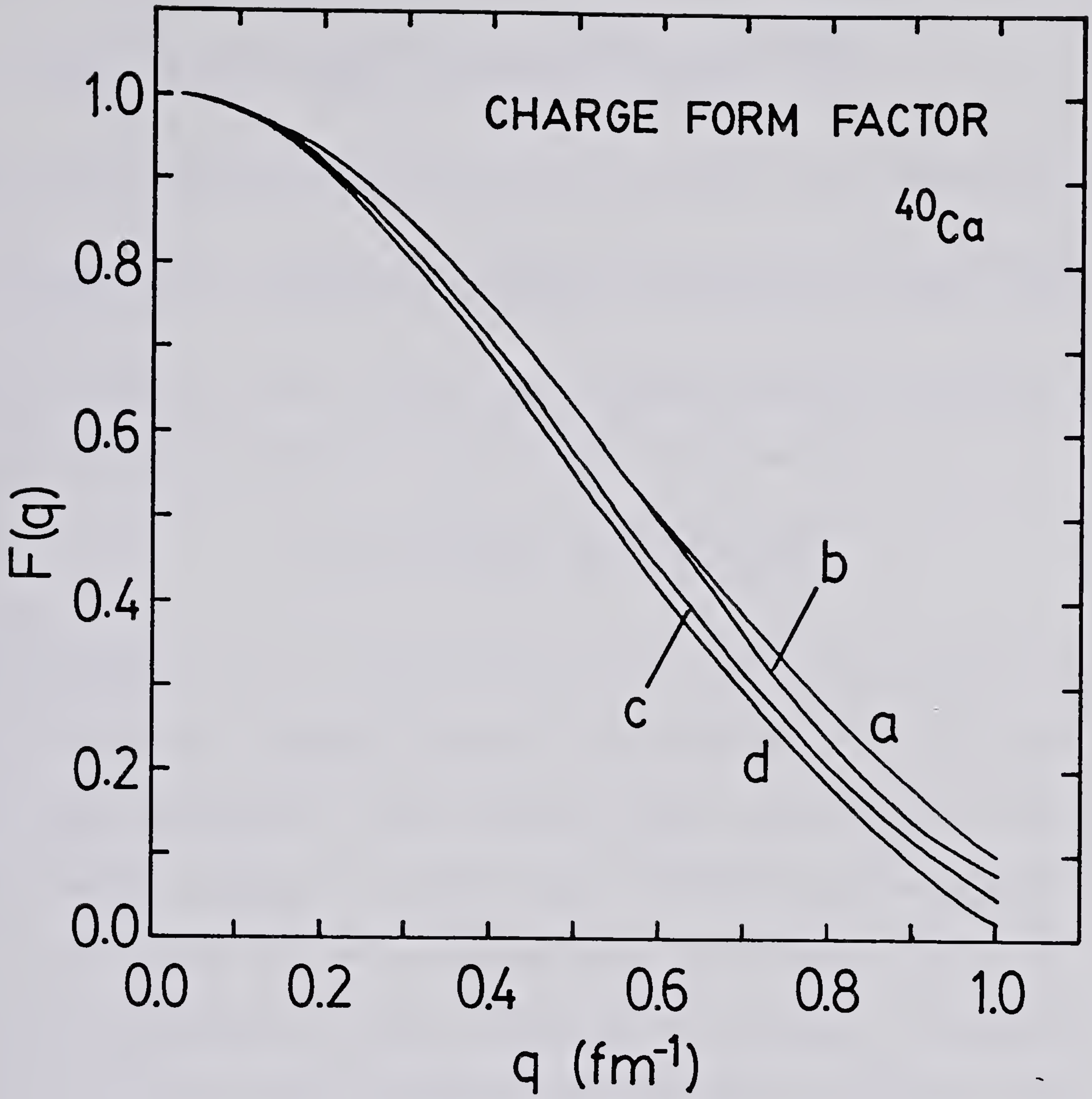

Fig. 14. Elastic scattering charge form factor for ^{40}Ca
a - CHO model using the SBL wave function, b=1.70 fm
b - experimental determination of Bellicard et al (Be 67)
c - CHO model using simple wave function, b=2.03 fm
d - CHO model using SBL wave function, b=1.90 fm.

expression for the form factor in the simple harmonic oscillator model is :

$$|F(q)|^2 = (1-0.25\eta^2+0.0125\eta^4)^2 \exp(-\eta^2/2) \qquad 5\text{-}8$$

with η=qb, b=2.03 fm, and using the SBL wave function is :

$$|F(q)|^2 = (1-0.346\eta^2+0.0386\eta^4-0.00416\eta^6)^2 \exp(-\eta^2/2) \qquad 5\text{-}9$$

with b=1.90 fm. The region of three-momentum transfer p-n of interest for radiative muon capture in ^{40}Ca is :

$$0 \leq |\eta/b| \leq 0.56\ \text{fm}^{-1} \qquad 5\text{-}10$$

From figure 14 it is seen that with the conventional choices for b both the simple harmonic oscillator and SBL wave functions predict a form factor which is slightly below experiment. Figure 15 shows the absolute photon spectrum computed using SBL wave functions, and figure 16, the relative spectrum. Since the means of fixing b employed by Shao et al (Sh 73) involved considerations based on ^{16}O and not ^{40}Ca, it was decided in addition to fix b for the SBL wave function by fitting the electromagnetic charge form factor over the range $0 \leq |\eta/b| \leq 0.6$ fm^{-1}. The fit gave b=1.70 fm, and the associated absolute and relative photon spectra also appear in figures 15 and 16. It should be noted that although the absolute spectrum is decreased, the relative spectrum is very nearly the same as for b=1.90 fm. In fact for the CHO model all relative spectra calculated here using a fixed set of parameters but different wave functions, ie.

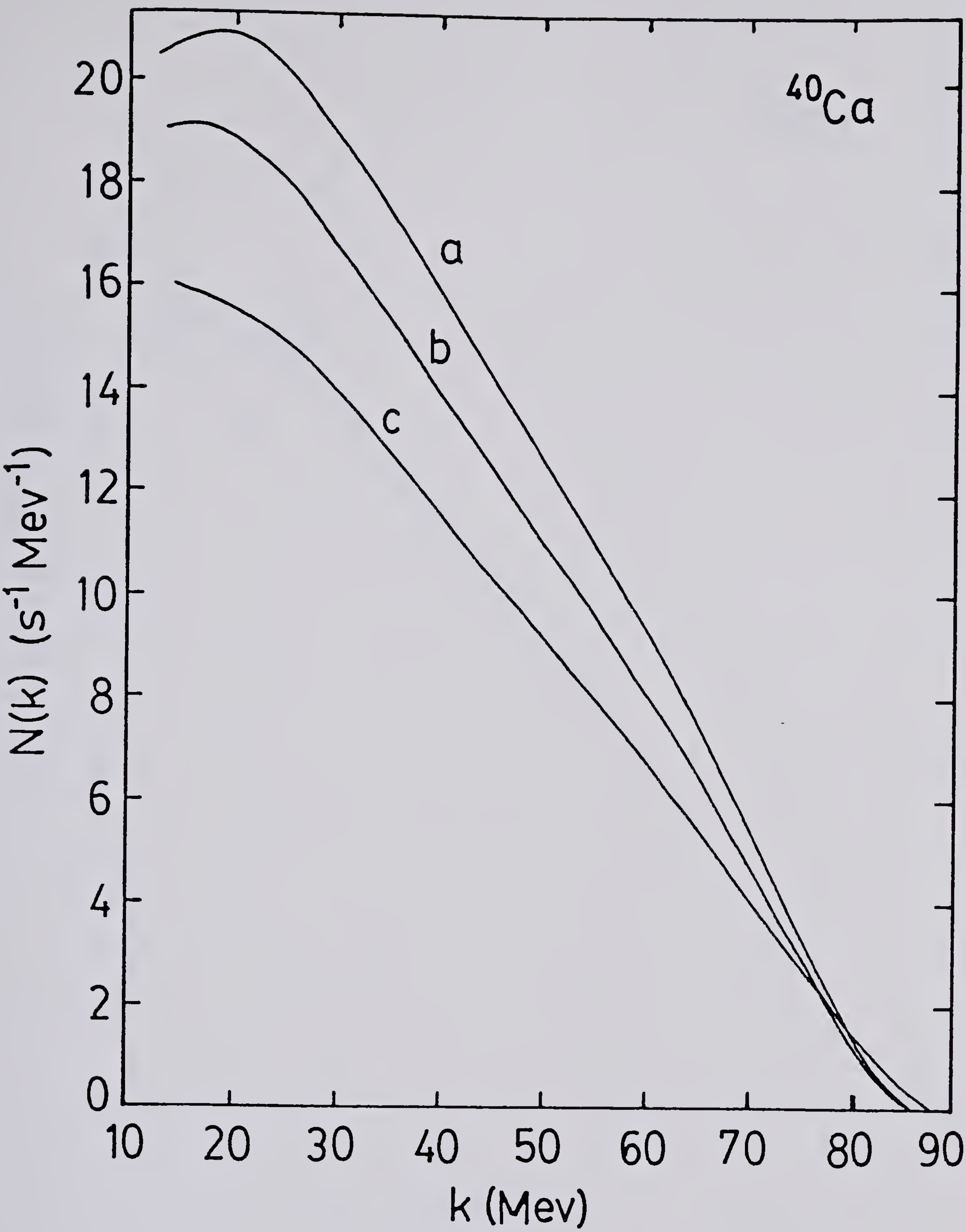

Fig. 15. Photon spectrum for ^{40}Ca calculated using SBL wave function

a - CHO model, b=1.90 fm
b - CHO model, b=1.70 fm
c - GDR model, b=1.70 fm.

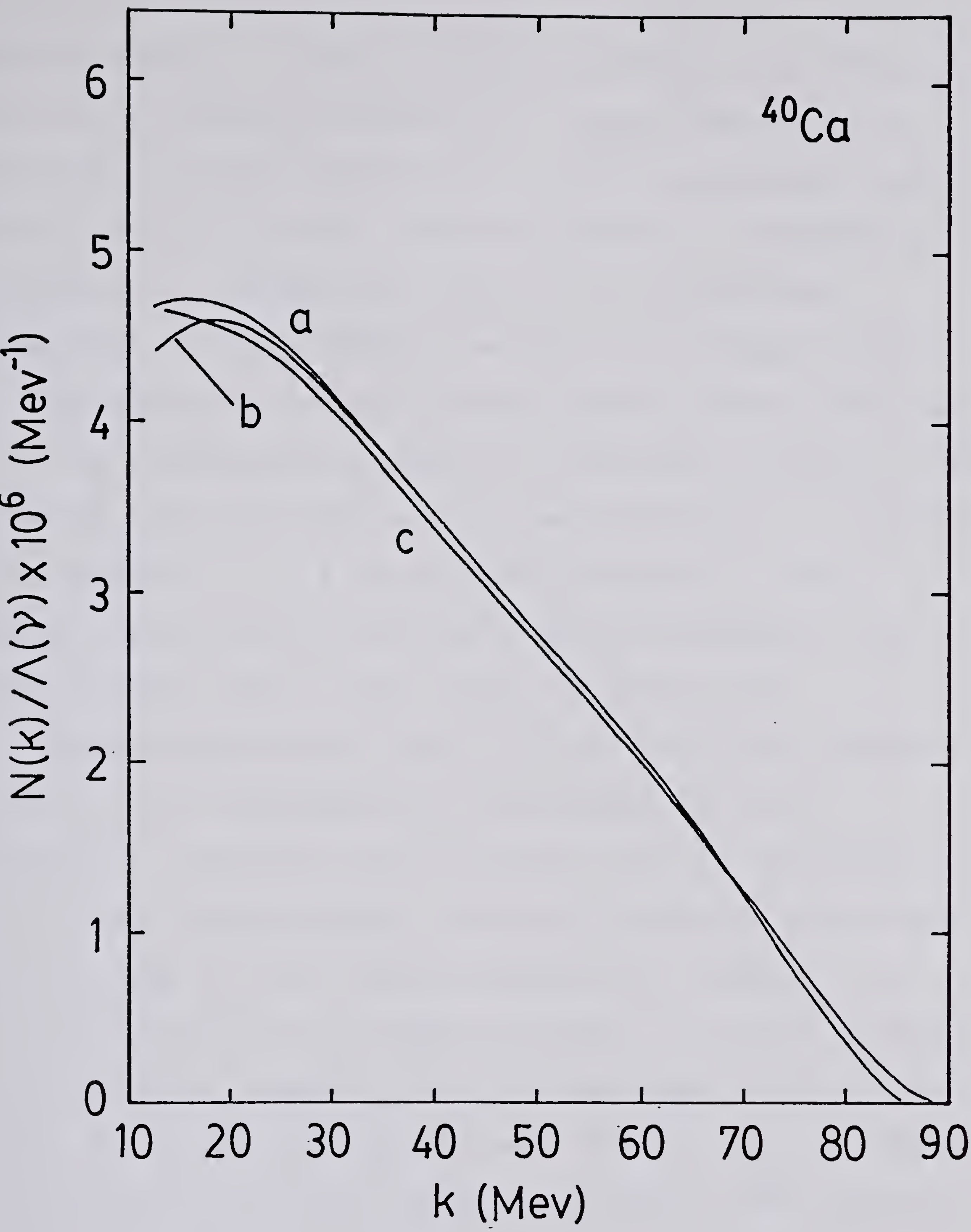

Fig. 16. Relative photon spectrum for ^{40}Ca calculated using the SBL wave function. The ordinary capture rate Λ is also given.
a - CHO model, b=1.70 fm ; $\Lambda = 4.05 \times 10^6$/s
b - CHO model, b=1.90 fm ; $\Lambda = 4.63 \times 10^6$/s
c - GDR model, b=1.70 fm ; $\Lambda = 3.44 \times 10^6$/s.

either the simple harmonic oscillator or SBL wave function for several different choices of b, are in quantitative agreement to within 5%. Thus one is able to conclude that in closure calculations the relative spectrum is insensitive to the choice of nuclear wave function. This conclusion supports the similar findings of Rood and Tolhoek (Ro 65).

The variation of the relative photon spectrum with k_m/ν is illustrated in figure 17. Although k_m can be roughly estimated from the experimental photon spectrum, in practice it is difficult to fix k_m accurately enough to obtain precise information on the weak coupling constants. This point is dealt with in more detail in Chapter VI.

Having examined the sensitivity of the photon spectrum to most of the parameters, we now present our best theoretical prediction for the photon spectrum calculated in the various nuclear models, assuming the Goldberger-Treiman value g_P=7xg_A for the induced pseudoscalar coupling. The other weak couplings have been assigned the current values in the standard parameter list. Our previous consideration of the ^{40}Ca charge form factor has indicated that the values b=2.03 fm for the simple harmonic oscillator wave function and b=1.70 fm for the SBL wave function are appropriate. The average energy parameters determined from the sum rule analysis carried out in Chapter VI are ν=79 Mev, k_m=84 Mev. For the GDR model the new photoabsorption data of Ahrens has been used. Figures 18 and 19 show the relative photon spectrum in the different models using these best

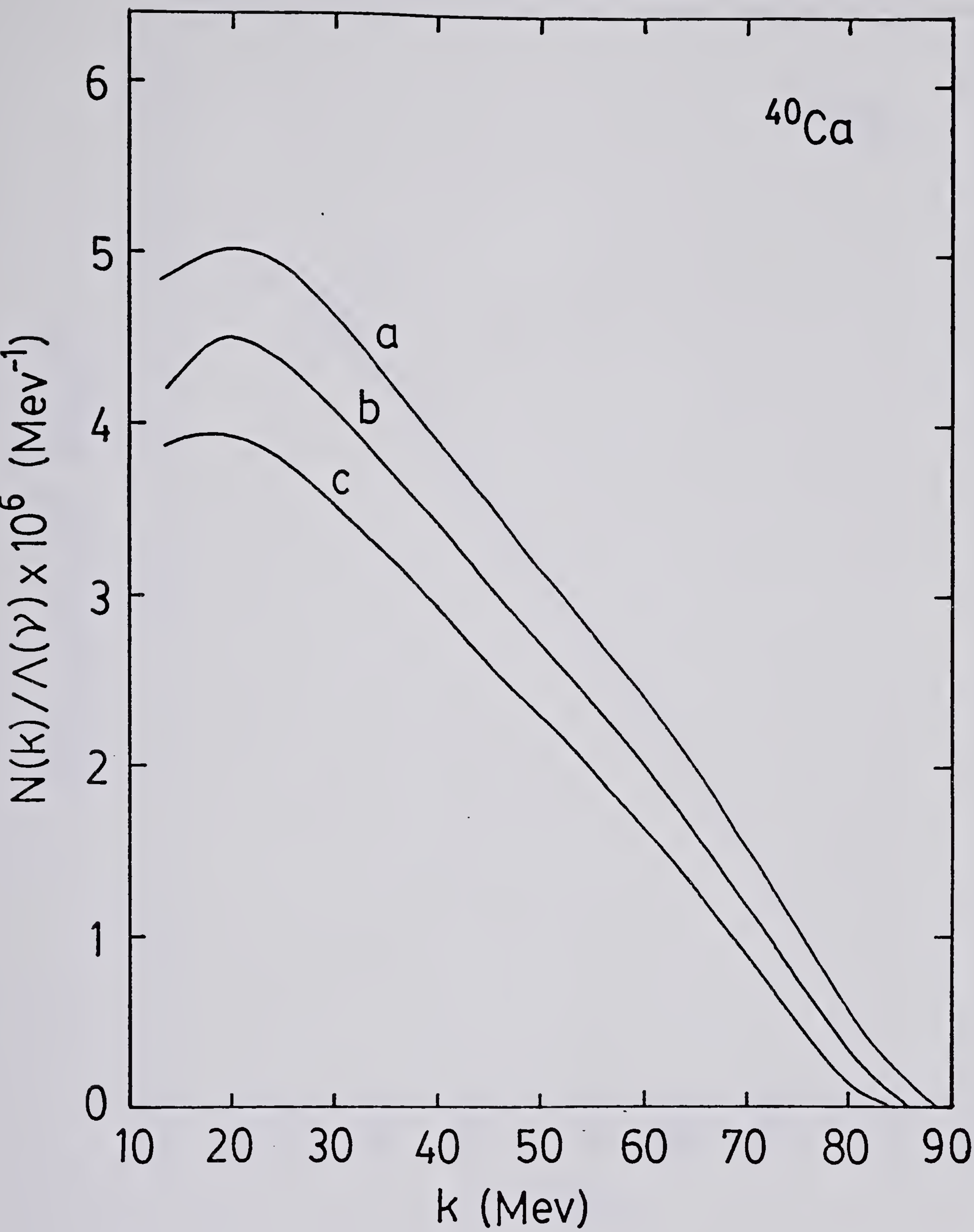

Fig. 17. Effect of varying the ratio k_m/ν on the relative photon spectrum for ^{40}Ca. The average neutrino energy was fixed at ν =85 Mev. The ordinary capture rate is $\Lambda = 4.36 \times 10^6$/s.

a - $k_m/\nu = 1.06$
b - $k_m/\nu = 1.03$
c - $k_m/\nu = 1.00$

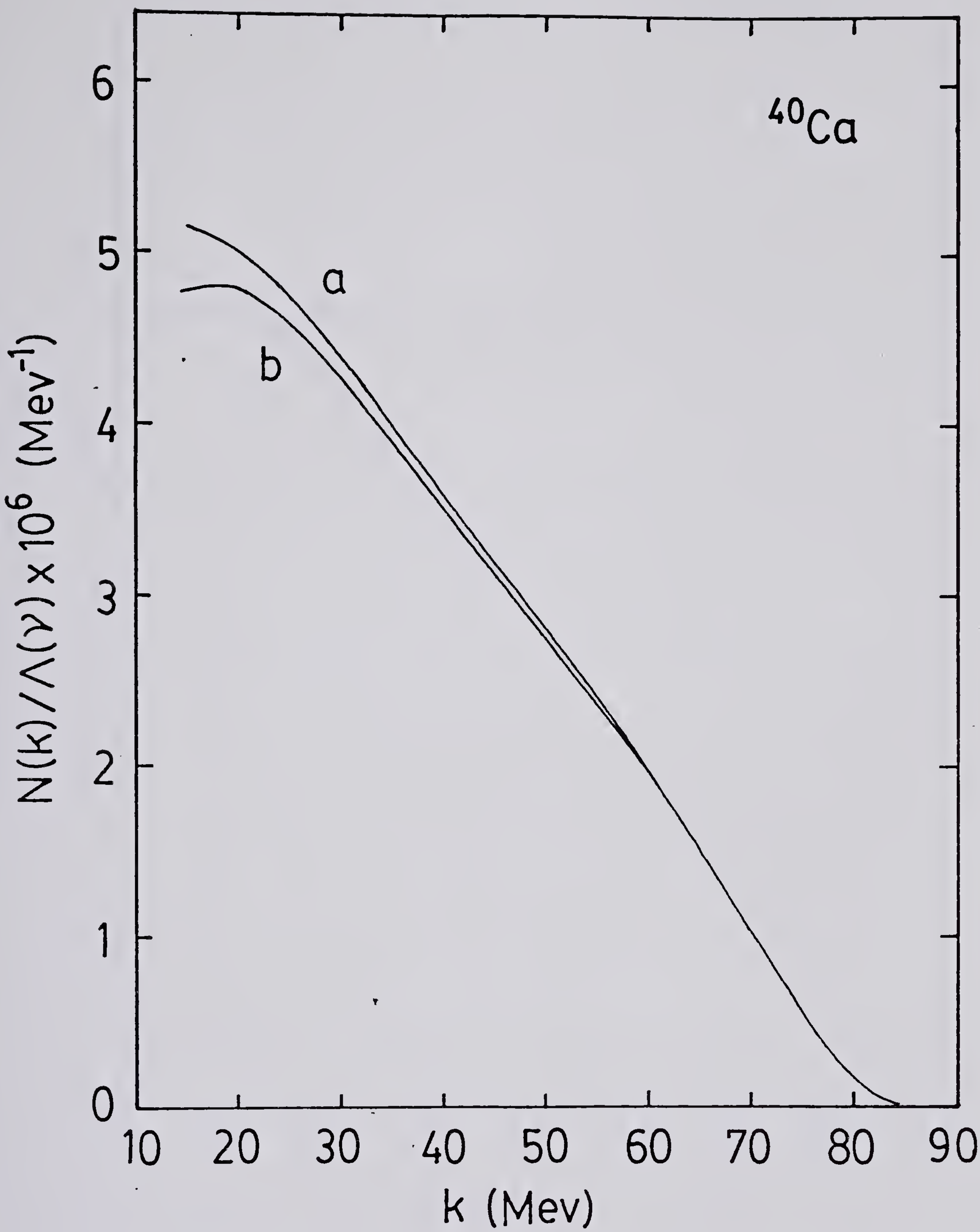

Fig. 18. ^{40}Ca relative photon spectrum for the best parameter choices
$\nu = 79$ Mev, $k_m = 84$ Mev, calculated in the CHC model. The ordinary capture rate Λ is also given. $\Lambda_{exp} = (2.29 \pm 0.06) \times 10^6$/s. (Ha 77)
a - SBL wave function; $\Lambda = 3.13 \times 10^6$/s
b - simple wave function; $\Lambda = 3.37 \times 10^6$/s.

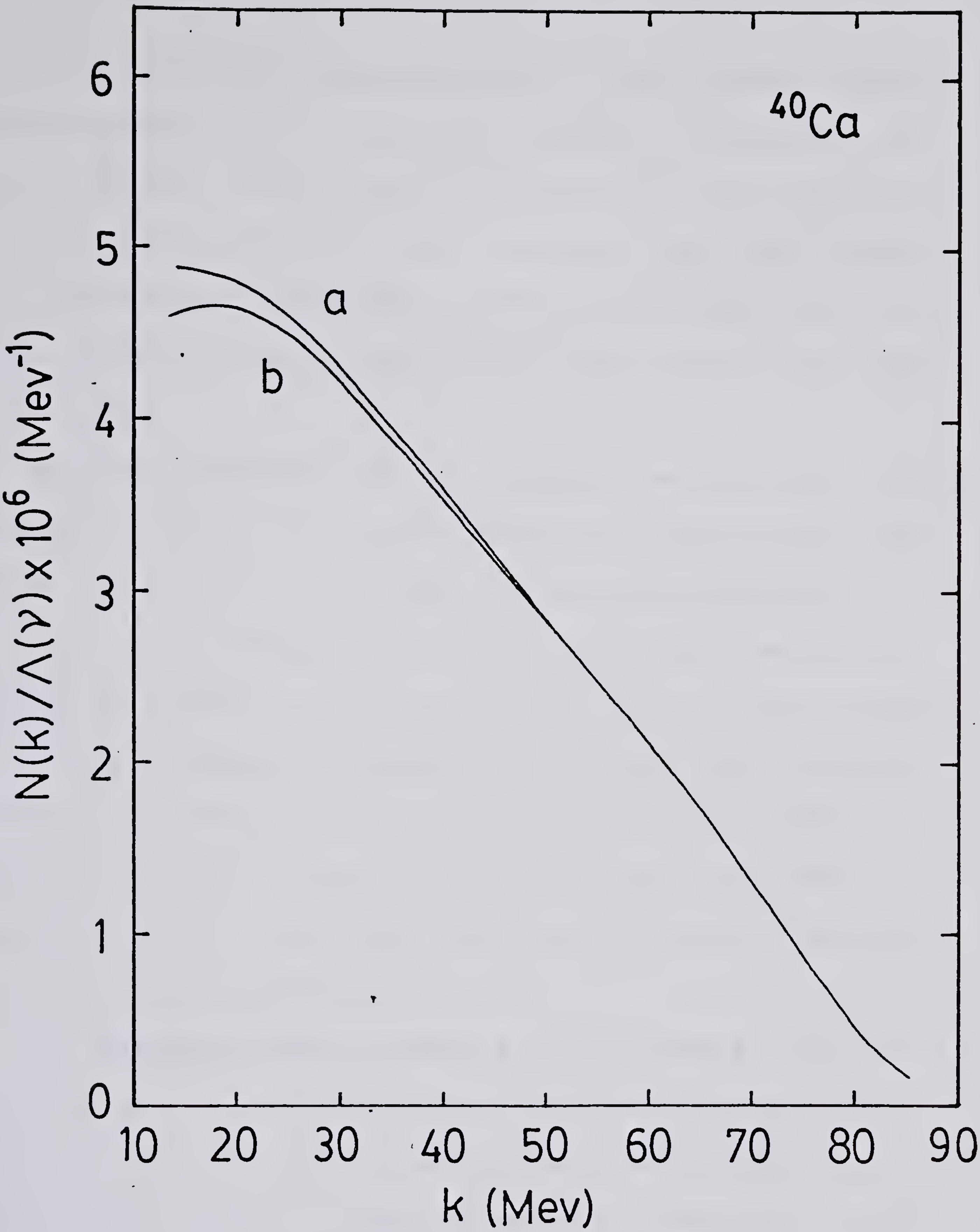

Fig. 19. ^{40}Ca relative photon spectrum for the best parameter choices
ν=79 Mev, k_m=84 Mev, calculated in the GDR model.
The ordinary capture rate Λ is also given. Λ_{exp} =(2.29±0.06)x10^6/s. (Ha 77)
a - SBL wave function; Λ=3.02x10^6/s
b - simple wave function; Λ=3.03x10^6/s.

parameters. Ordinary capture rates are also given. Compared with experiment, all of the rates come out too high by as much as 50% for the CHO model ordinary rate and a factor of almost three for the GDR model radiative rate for k>60 Mev. In the best cases the ordinary rate is 30% high for the GDR model and the radiative rate is 60% high for the CHO model for k>60 Mev.

Recently evidence for the quenching of the axial vector strength by up to 25% in medium and heavy mass nuclei has emerged (Br 78, Os 79, To 79). For maximal quenching g_A=-0.9, g_P=-6.3=7xg_A. The corresponding relative photon spectrum is presented in figure 20 for the SBL wave function and the best parameters given above. [4] While the relative spectrum is changed only slightly, both the ordinary and radiative absolute rates are decreased some 60%. Such a large effect is in the right direction to achieve agreement with experimentally measured rates.

The relative photon spectrum is especially sensitive to the value of the induced pseudoscalar coupling g_P, hence potentially can be used to obtain information about g_P. A complication arises in that the relative spectrum is also sensitive to the average maximum photon energy k_m which cannot be accurately fixed in an a priori manner. The usual approach to this difficulty, which will be followed here, is to take both g_P and k_m as adjustable parameters in fitting

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[4] Note that the parameters ν=79 Mev, k_m=84 Mev were determined assuming no quenching, and that a revised sum rule analysis incorporating quenching might yield different values of ν and k_m.

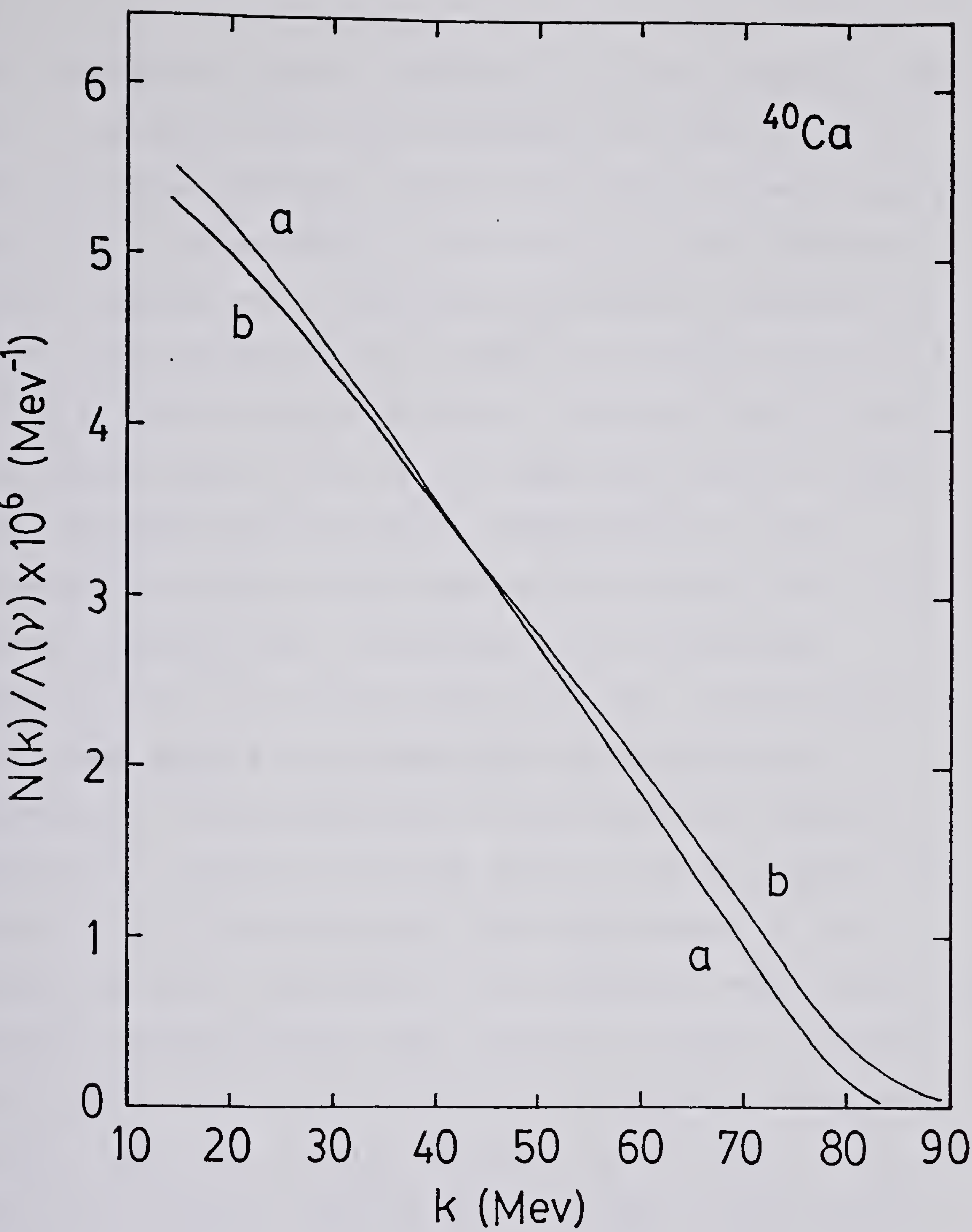


Fig. 20. ^{40}Ca relative photon spectrum for the axial vector strength quenched 25%. The SBL wave function and best parameter choices $\gamma = 79$ Mev, $k_m = 84$ Mev were used with $g_A = -0.9$, $g_P = -6.3 = 7 x g_A$. The ordinary capture rate Λ is also given
a - CHO model; $\Lambda = 2.05 x 10^6$/s
b - GDR model; $\Lambda = 1.98 x 10^6$/s.

the experimental photon spectrum. The fixed parameters were set to the best values just discussed. Figures 21 and 22 show two representative fits to the relative photon spectrum for $57 \leq k \leq 90$ Mev measured by Hart et al (Ha 77), using our $C(1/m^2)$ squared matrix element. The figures correspond to cases 1 and 2 respectively in table I. Complete results of all fits done are given in table I. Similar kinds of fits were done by Hart et al (Ha 77) using the theory of Rood, Yano, and Yano (Ro 74) which includes effects on the intermediate states of the muon and the nucleon due to the nuclear Coulomb field. Their best fit, done setting $k_m/\nu=1.02$, $g_A=-1.24$, gave $k_m=86.5\pm1.9$ Mev, $g_P=-8.13\pm2.00$, the fitted value for g_P apparently agreeing with the Goldberger-Treiman prediction of $g_P=7 x g_A=-8.75$. Absolute ordinary and radiative capture rates in the RYY theory are however still some 80% higher than experiment. As the results in table I illustrate, the quenching of g_A affects the relative photon spectrum only slightly; however as pointed out earlier the absolute rates can be decreased by as much as 60%. Thus if g_A is truly quenched on the order of 25% in ^{40}Ca, the RYY theory may be able to predict both absolute and relative radiative capture rates using a value for g_P consistent with the Goldberger-Treiman estimate.

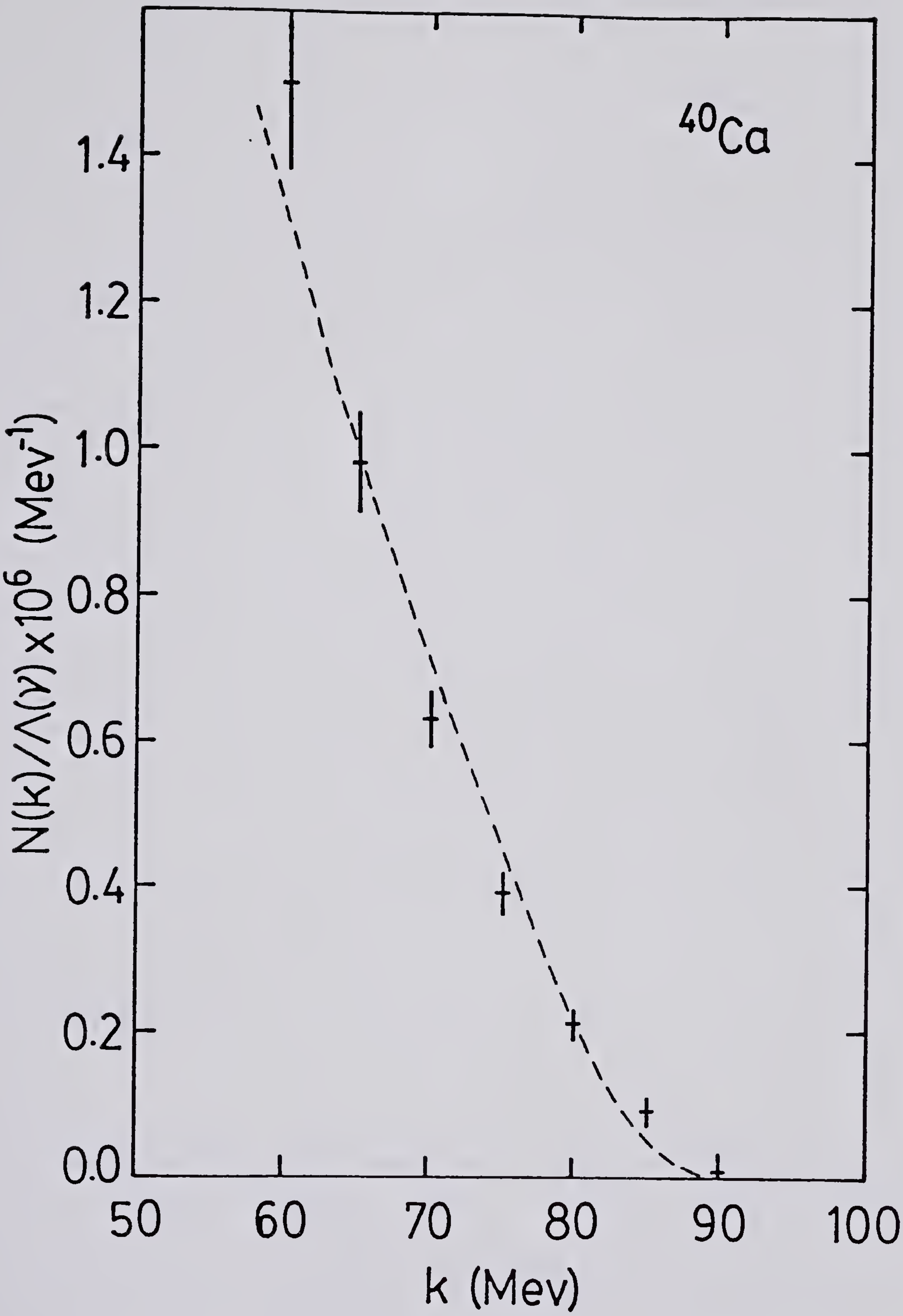


Fig. 21. Two-parameter fit to the relative photon spectrum of Hart et al (Ha 77) using the CHO model with the SHO wave function and $k_m/\gamma=1.06$. Results for the fitted parameters are $k_m=89.1\pm0.9$ Mev, $g_P=(-2.8\pm4.8)\,g_A$. The ordinary capture rate is $\Lambda=5.3\times10^6$/s.

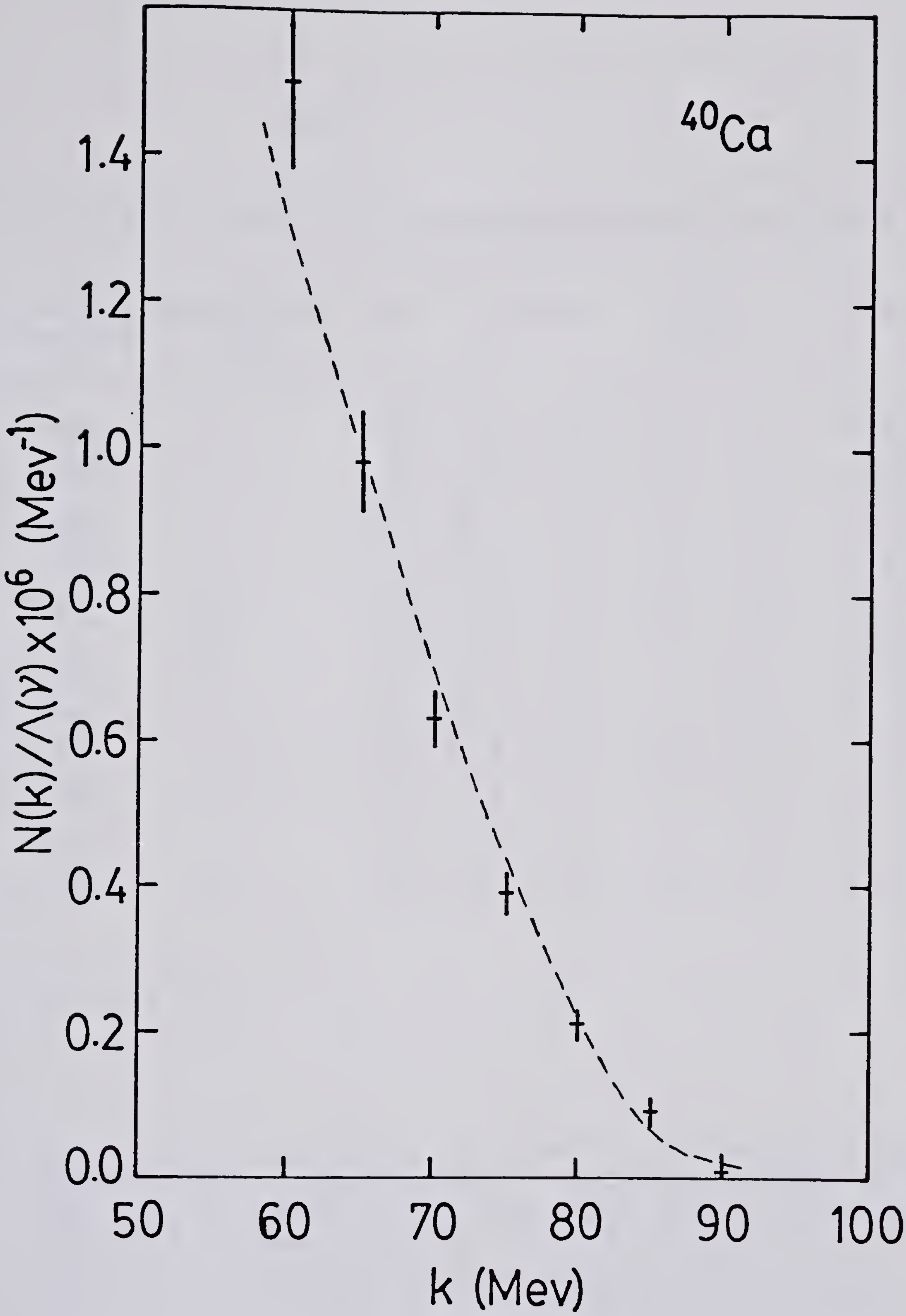


Fig. 22. Two-parameter fit to the relative photon spectrum of Hart et al (Ha 77) using the GDR model with the SHO wave function and k_m/γ=1.06. Results for the fitted parameters are k_m=87.0±3.0 Mev, $g_P = (-0.3\pm2.0)\, g_A$. The ordinary capture rate is Λ =3.9x10^6/s.

Table I

Fits to the relative photon spectrum measured by Hart et al (Ha 77)

Case	Model	Wave function	g_A quenched 25%	k_m/ν	k_m(Mev)	g_P/g_A	$10^{-6}\Lambda(s^{-1})$
1	CHO	SHO	no	1.06	89.1 ± 0.9	-2.8 ± 4.8	5.29
2	GDR	SHO	no	1.06	87.0 ± 3.0	-0.3 ± 2.0	3.89
3	CHO	SBL	no	1.06	89.1 ± 1.0	-3.5 ± 4.5	5.01
4	GDR	SBL	no	1.06	86.4 ± 3.6	-0.2 ± 1.9	3.75
5	CHO	SBL	yes	1.06	90.5 ± 2.4	-7.7 ± 6.3	3.63
6	GDR	SBL	yes	1.06	86.4 ± 3.2	-1.4 ± 3.2	2.45
7	CHO	SHO	no	1.03	89.4 ± 1.4	-1.1 ± 3.0	5.59
8	GDR	SHO	no	1.03	86.9 ± 3.1	0.8 ± 1.5	4.05
9	CHO	SHO	no	1.00	89.4 ± 1.6	0.7 ± 2.3	5.99
10	GDR	SHO	no	1.00	86.8 ± 3.0	1.8 ± 1.2	4.24
11†	CHO	SHO	no	1.06	89.1 ± 0.9	0.8 ± 1.0	5.01

†without any $O(1/m^2)$ terms

Table I. Results for k_m and g_P obtained from two-parameter fits to the relative photon spectrum of Hart et al (Ha 77). The other weak couplings were assigned the standard values referred to in the text; the value for the axial vector strength quenched 25% was $g_A=-0.9$. Also given is the ordinary muon capture rate Λ.

C. Photon Asymmetry

If the incoming muon in radiative capture is polarized, then there will exist a measureable correlation between the direction of the emitted photons and the muon polarization vector. In particular the directional distribution of the photons can be expressed (Ro 65) :

$$\vec{N}(x) = \frac{1}{4\pi} N(x) \left(1 + \alpha\, \vec{\xi} \cdot \hat{k}\right) \tag{5-11}$$

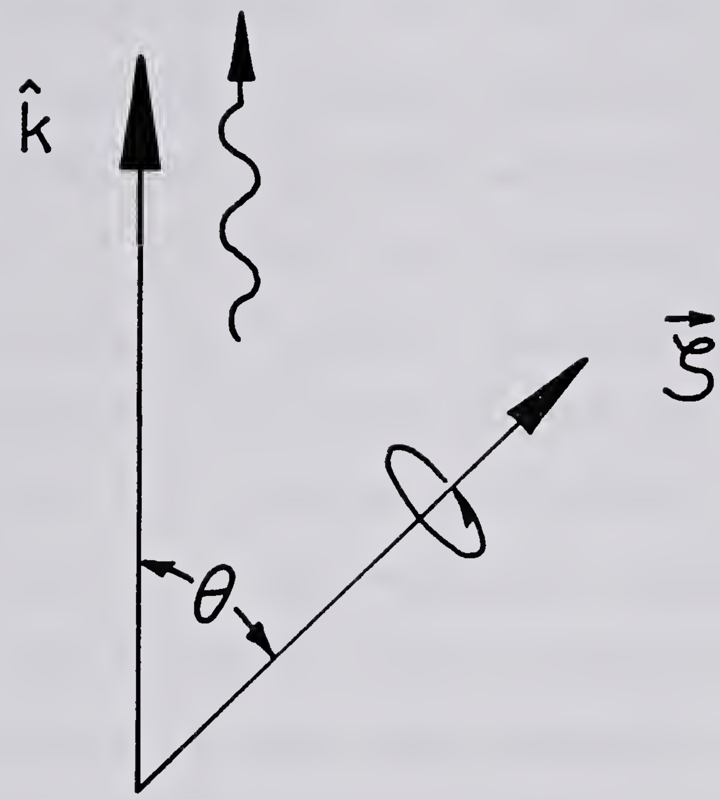
where N(x) is the photon spectrum for unpolarized muons. The photon asymmetry is depicted schematically in figure 23. Fearing (Fe 75) has shown that for the standard theory of radiative muon capture (Ro 65) including all weak couplings except g_S and all Feynman diagrams shown in figure 1, the photon asymmetry satisfies :

$$\alpha = 1 + O\left(\frac{1}{m^2}\right) \tag{5-12}$$

where m is the nucleon mass. Thus to first order the entire correction to $\alpha \neq 1$ comes from the $O(1/m^2)$ terms in the squared matrix element. One can make the following observations :

1. the leading O(1) terms in the squared matrix element all originate from the muon radiating diagram
2. as pointed out by Fearing (Fe 75), the muon radiating diagram and its interference with all other diagrams contribute only to $\alpha = 1$ to all orders in 1/m and for all six couplings.

Hence the first order $O(1/m^2)$ correction to $\alpha \neq 1$ comes



$$\vec{N}(x) = 1/4\pi \times N(x)[1+\alpha|\vec{\xi}|\cos\theta]$$

Fig. 23. Pictorial representation of the angle θ entering into the expression for the photon spectrum in the case of polarized muons.

entirely from the square of $O(1/m)$ terms in the effective Hamiltonian originating from diagrams other than the muon radiating one. Similarly the second order $O(1/m^3)$ correction to $\alpha\neq 1$ comes from the product of $O(1/m)$ and $O(1/m^2)$ terms in the effective Hamiltonian originating from diagrams other than the muon radiating one. Thus one is able to compute the photon asymmetry to $O(1/m^3)$ from a knowledge of the effective Hamiltonian to $O(1/m^2)$. The calculated asymmetry is shown in figure 24. Note that the $O(1/m^3)$ correction amounts to at most a 3% drop in the $O(1/m^2)$ asymmetry, a decrease consistent with the collective magnitude of the $O(1/m^3)$ terms. For the giant dipole resonance model the asymmetry had not in the past been computed. This entirely new result appears in figure 25. Over the experimentally observed region $50<k<90$ Mev the curve is only a few percent lower than the harmonic oscillator result. Apparently the parts of the squared matrix element independent of, and depending on $\vec{\zeta}$ respectively, change in a similar fashion when the nuclear matrix elements are changed, indicating that the asymmetry depends little on nuclear particulars.

It should be emphasized that the above discussion of the photon asymmetry has been limited to calculations done within the framework of the standard theory. With the inclusion of Coulomb and strong nuclear potentials, the theorem proved by Fearing (Fe 75) is no longer generally valid (see Appendix F for a detailed discussion of this point) so that the presence of additional interactions can

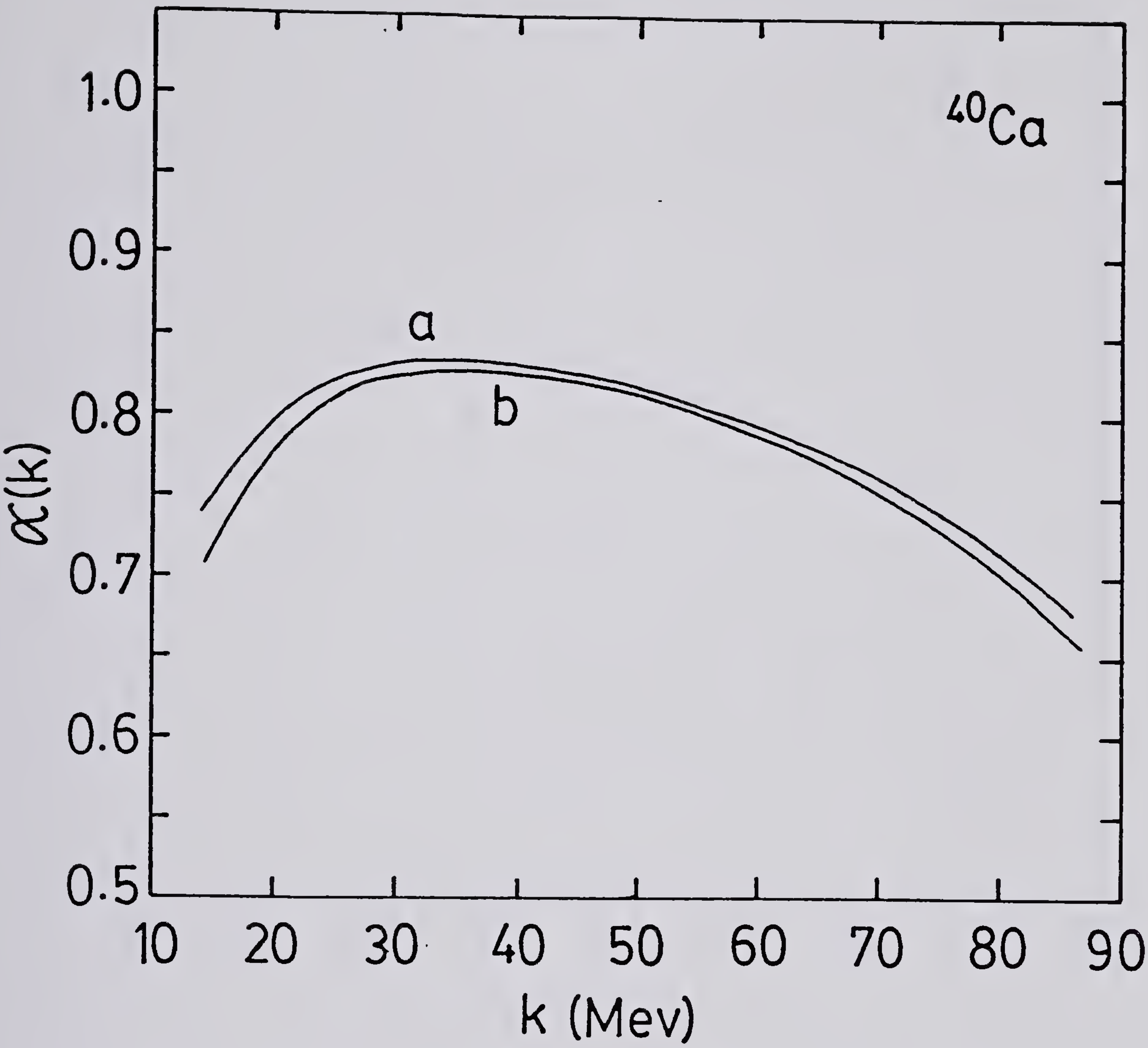


Fig. 24. Photon asymmetry for ^{40}Ca calculated in the CHO model
a - $O(1/m^2)$ result
b - $O(1/m^3)$ result

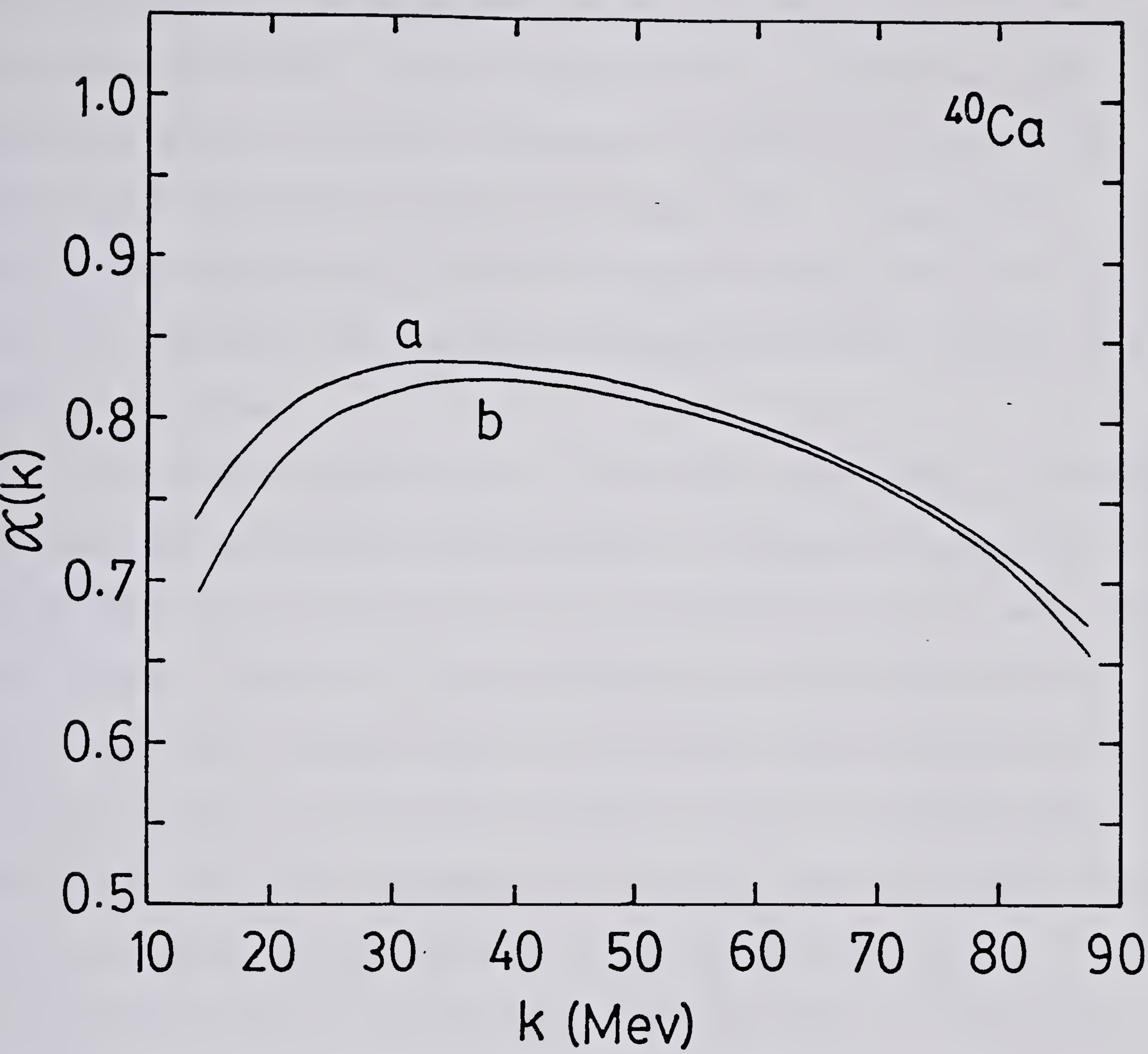


Fig. 25. Photon asymmetry for ^{40}Ca calculated in the GDR model to $O(1/m^2)$
a - CHO model $O(1/m^2)$ result
b - GDR model

affect the asymmetry in lower orders of $1/m$ than the second. An indication that the asymmetry is just so affected is provided by the work of Rood et al (Ro 74), who considered Coulomb and strong nuclear potentials in radiative muon capture. Their computed asymmetry is slightly higher than the standard theory result, so that while it appears Fearing's asymmetry theorem no longer holds, it also seems that the effects of exterior potentials on the asymmetry are small for ^{40}Ca.

The photon asymmetry is virtually unaffected by changes in some of the variables on which it depends. Among the variables in this category are the oscillator parameter of the simple shell model wave function or the Hartree-Fock wave function of SBL where it is used, the average maximum photon energy k_m, and the velocity terms. In summary the asymmetry does not depend much on the nuclear physics. This point is well illustrated by the fact that our best predictions for the asymmetry, corresponding to the best parameter choices made in the previous section for the photon spectrum, are essentially indistinguishable unless the distinction is made between the CHO and GDR models. Thus our best CHO model predictions both for the simple harmonic oscillator and SBL wave function differ from the asymmetry in figure 24 by less than 1%, and similarly for the GDR model asymmetry in figure 25.

The asymmetry does depend to a considerable degree however on the values of the weak coupling constants. In

examining this dependence, the asymmetry will be calculated to $O(1/m^2)$ in the closure-harmonic oscillator model using our standard set of parameters.[5] Figure 26 shows the asymmetry for $g_V=0$; $g_A=g_P=0$; $g_A=-0.9$, $g_P=-6.3=7xg_A$; illustrating, in view of the g_P presented below, that α exhibits strong g_A dependence. The latter value of g_A corresponds to the axial vector strength quenched 25%. Figure 27 depicts the asymmetry calculated for non-zero values of g_S and g_T, $g_S=\pm1$, $g_T=\pm g_M$. In these cases α varies at most ±10%. Finally figure 28 shows the photon asymmetry for values of the induced pseudoscalar coupling $g_P=0$, $4xg_A$, and $10xg_A$. With $g_P=0$, α assumes a value close to 1, while as g_P takes on increasingly negative values, α decreases fairly rapidly. Thus experimental measurements of the photon asymmetry may prove useful in fixing g_P, provided the other couplings can be independently and accurately determined. It would seem g_A presents the biggest problem in this regard. Hart et al (Ha 77) have measured α for photon energies $57<k<90$ Mev, obtaining an average value $\bar{\alpha}=+0.90\pm0.50$. While this result is in agreement with the standard theory and with our results, a more precise measurement of the variation of α with k is clearly desireable.

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[5] As fig. 24 seems to indicate that the $O(1/m^3)$ terms change the asymmetry by a few percent at most, these terms have been omitted here in the interest of computational expediency. It has been verified for the cases $g_V=0$, $g_A=g_P=0$ discussed below that inclusion of the $O(1/m^3)$ terms indeed affects the asymmetry by no more than 3%.

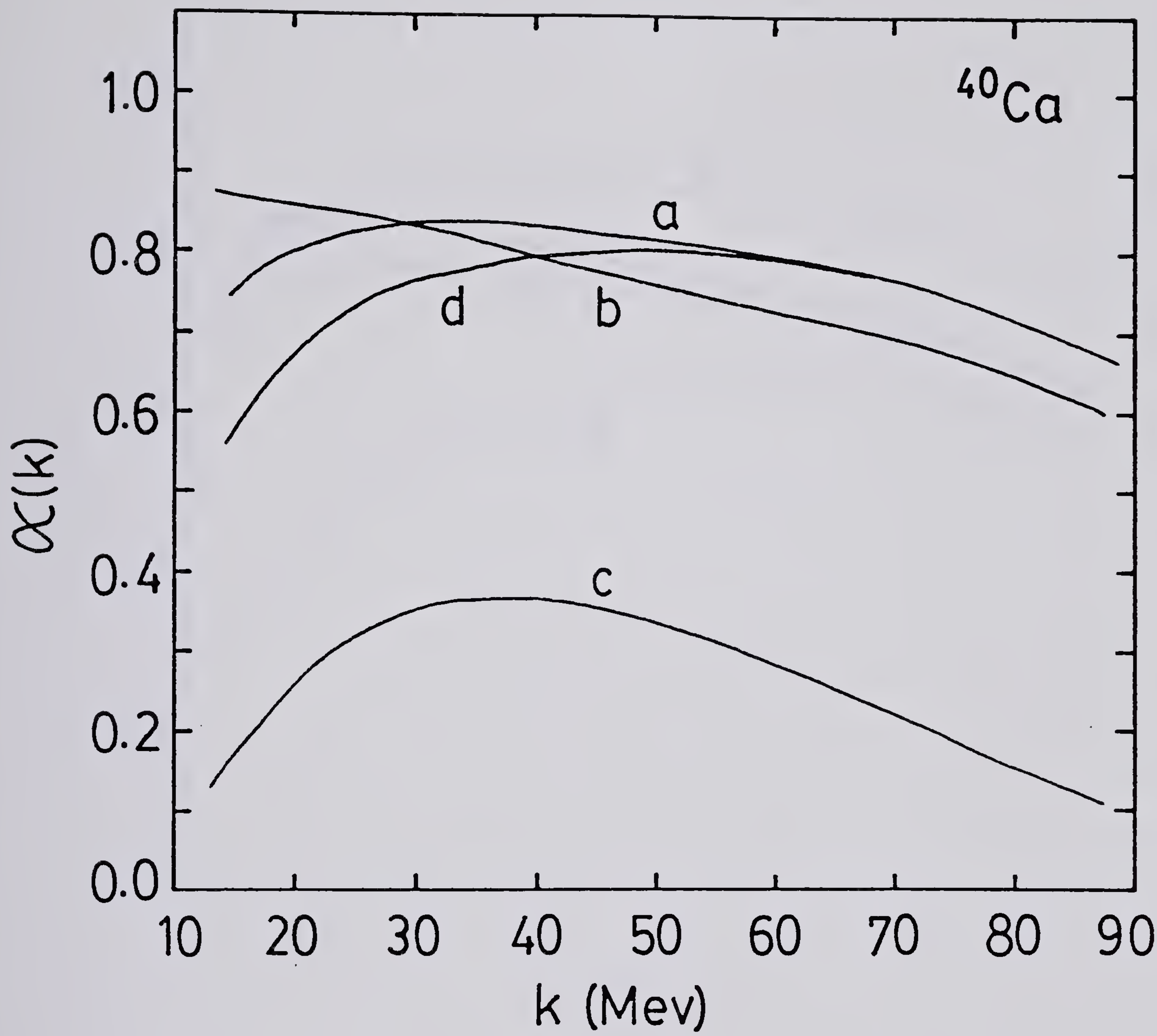

Fig. 26. Effect on the photon asymmetry for ^{40}Ca of the weak couplings g_V and g_A, calculated in the CHO model
a - complete $O(1/m^2)$ result
b - $g_V=0$
c - $g_A=0$.
d - $g_A=-0.9$, $g_P=7xg_A$ (g_A quenched 25%).

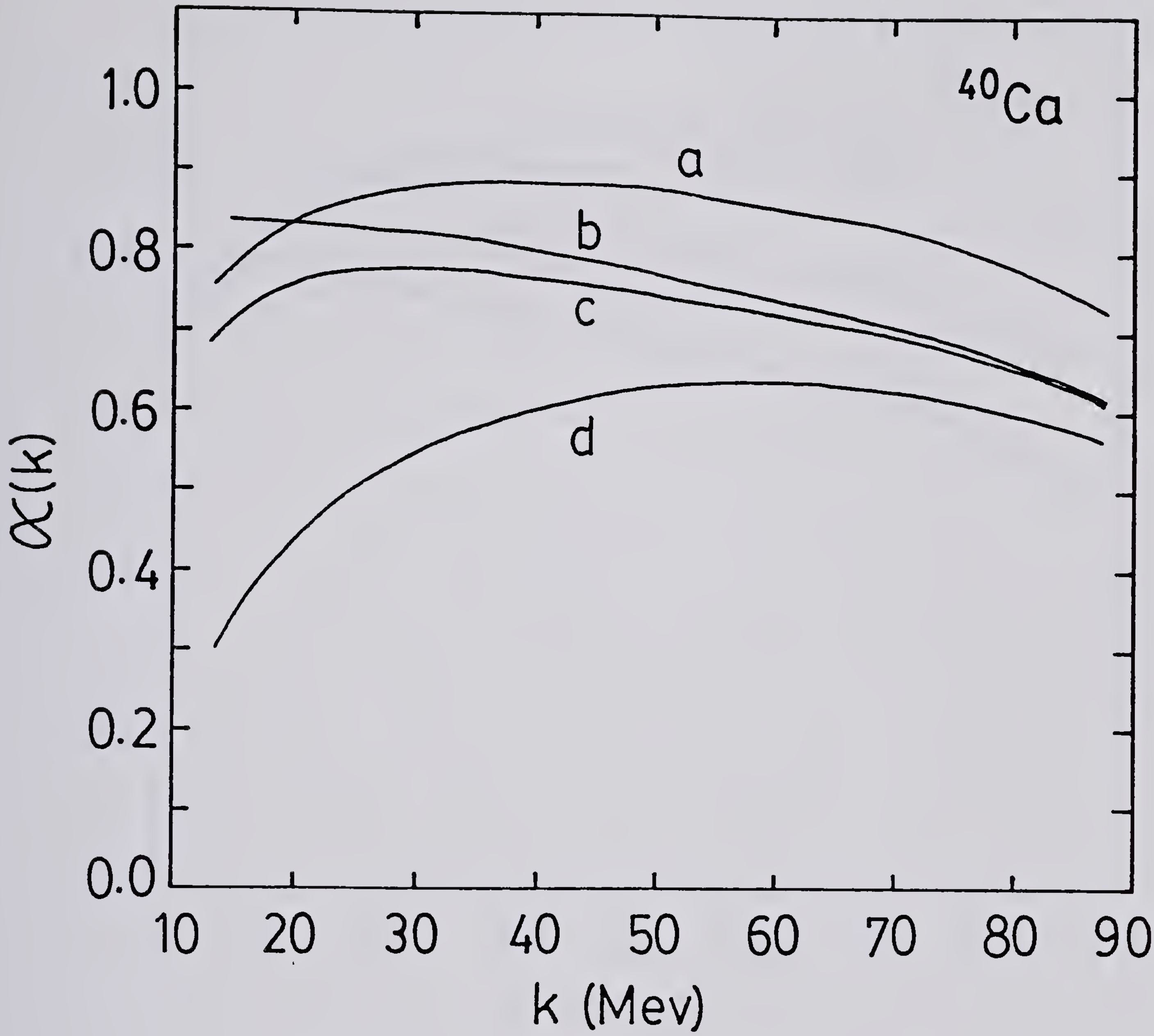

Fig. 27. Effect on the photon asymmetry for ^{40}Ca of varying g_s and g_T, calculated in the CHO model

a - $g_T = -g_M = -3.7$
b - $g_s = -1.0$
c - $g_T = +g_M = +3.7$
d - $g_s = +1.0$

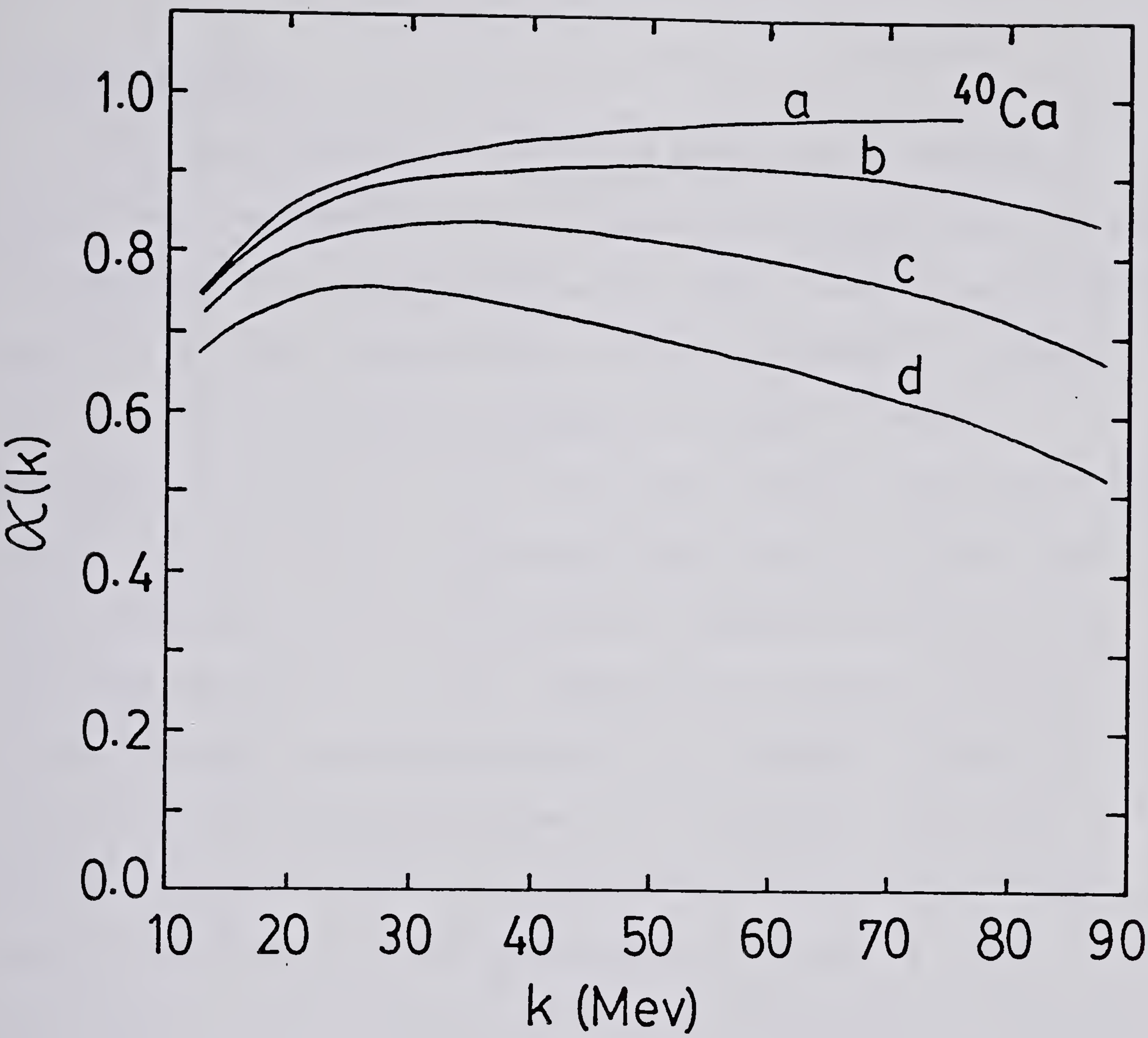

Fig. 28. Effect on the photon asymmetry for ^{40}Ca of varying g_P, calculated in the CHO model
a - $g_P=0.0$
b - $g_P=4xg_A=-5.0$
c - $g_P=7xg_A=-8.75$
d - $g_P=10xg_A=-12.5$

VI.

A MODIFIED CLOSURE CALCULATION

A. Introduction

The calculation of radiative muon capture rates in nuclei has been pursued by a number of authors with the aim of extracting the magnitude of the weak induced pseudoscalar coupling g_p from measurements of the differential photon spectrum or the photon asymmetry. In most of these calculations (e.g. Pr 59, Ro 65) the closure approximation on the final nuclear states has been used to express the rate in terms of the ground state expectation value of a two-body operator. In this approach an average maximum photon energy k_m corresponding to an average excitation energy of the residual nucleus is introduced. The rate depends very strongly on k_m and hence uncertainty in k_m poses a problem when one attempts to extract g_p from experiment. The situation is entirely analogous to that for ordinary muon capture, where the result of a closure approximation calculation is very sensitive to the choice of the average neutrino energy ν.

Recent calculations (Do 72, Be 73A, Be 73C, Go 74, Ko 76) of ordinary muon capture rates employing sum rules have succeeded in largely eliminating the ν dependence of the rate. In particular, Bernabeu (Be 73A) has shown that a first order expansion of the capture rate about ν gives a result which is essentially independent of the specific

value of γ within a range of plausible values.

In the present chapter these ideas are extended to radiative muon capture and it is shown that a first order expansion of the expression for the photon spectrum about k_m gives a corrected expression for the spectrum which is much less dependent on the specific value of k_m used than the usual closure result.

In Sec. B a corrected expression for the differential photon spectrum is derived. As for ordinary capture, two additional terms arise besides the usual closure term. One can be calculated from the closure term and the other one is evaluated using a generalized Thomas-Reiche-Kuhn (TRK) sum rule (Er 70).

Several further corrections are considered which were not included in earlier calculations of ordinary capture. In particular, the effect of the Coulomb energy difference between initial and final nuclear states is included in a simple way. Exchange effects are also included, albeit in a phenomenological way, by modifying the sum rule term by an overall factor which is obtained from a sum rule analysis of total photo-absorption cross sections.

In Sec. C the meaning of these average quantities k_m and γ is clarified and it is shown that by a simple extension of the sum rule technique, a consistency equation which allows an explicit calculation of average quantities related to k_m and γ can be obtained. Then as an example in Sec. D our results are applied to the closed shell nucleus

^{40}Ca, using a simple harmonic oscillator model, and finally a brief discussion of these results and our conclusions is presented.

B. Theory

In an approximation which neglects the "velocity terms", the differential photon spectrum for radiative muon capture can be written :

$$N(k) = (\alpha m_\mu G^2/4\pi^2)\,|\varphi_\mu|^2 \sum_{\lambda=\pm 1} I^2(k,\lambda) \tag{6-1}$$

Here φ_μ is the muon wave function, and the sum on λ is over the circular polarizations of the photon emitted with absolute value of momentum k. The function $I^2(k,\lambda)$ contains the nuclear matrix elements. If one assumes for radiative capture the relations given in Eq. 3-11, then :

$$\begin{aligned}\sum_\lambda I^2(k,\lambda) &= \int_{-1}^{1} dy \sum_{\bar{a}b} G_{ab}^2 \left\{ \frac{k(k_m^{ab}-k)^2}{m_\mu^3}\,\theta(k_m^{ab}-k)\,\Big|\langle b|\sum_j J_-(j)\,e^{-i\vec{s}_{ab}\cdot\vec{r}_j}|a\rangle\Big|^2 \right\} \\ &\equiv \int_{-1}^{1} dy \sum_{\bar{a}b} G_{ab}^2\, I_{ab}^2(k_m^{ab},k)\end{aligned} \tag{6-2}$$

In this expression $y = \hat{k}\cdot\hat{\nu}$ and $\vec{s}_{ab} = (\vec{k} + \vec{\nu})_{ab}$, with $\vec{\nu}_{ab}$, $\vec{k}_{ab}$ and k_m^{ab} respectively the neutrino momentum, the photon momentum, and the maximum photon energy corresponding to the transition $a \to b$. G_{ab}^2 is a function of k_m^{ab} , y, and the weak coupling constants, and the sum on $\bar{a}b$ denotes an average over initial and a sum over final nuclear states. The maximum energy available to the photon is given by :

$$k_m^{ab} = m_\mu - (m_N - m_P) - E_{BE} - (E_b - E_a)$$
$$\equiv E - (E_b - E_a) \qquad 6\text{-}3$$

in which E_{BE} is the muon binding energy and E_a, E_b are the energies of the nuclear states.

Since G^2_{ab} depends only weakly on k_m^{ab} through the combination $k_m^{ab}/2m \sim 0.05$, we replace k_m^{ab} in G^2_{ab} by an appropriate average value k_m, and define :

$$I_n(k) \equiv \int_{-1}^{1} dy\, y^n \sum_{\bar{a}b} I^2_{ab}(k_m^{ab}, k) \qquad 6\text{-}4$$

Thus the differential photon spectrum can be written :

$$N(k) = (\alpha m_\mu G^2/4\pi^2)\,|\varphi_\mu|^2 \sum_n C_n I_n(k) \qquad 6\text{-}5$$

The C_n , which are obtained by extracting the explicit y dependence from G^2_{ab} ,are functions of k_m and the weak couplings. When the I_n are evaluated in the closure-harmonic oscillator model, Eq. 6-5 gives an expression for the spectrum in closure approximation, $N(k_m,k)$, which exhibits strong dependence on the average maximum photon energy k_m, as will be seen in the next section.

In an attempt to remedy this situation, correction terms to the closure approximation were calculated using techniques analogous to those used by Bernabeu (Be 73A) for ordinary capture. Thus $I^2_{ab}(k_m^{ab}, k)$ was expanded to first order in k_m^{ab} about an average value k_m and a corrected expression $\tilde{I}^2_{ab}$ for which the k_m^{ab} dependence is explicit and linear was obtained.

$$\tilde{I}^2_{ab} = I^2_{ab}(k_m^{ab}, k)\Big|_{k_m^{ab}=k_m} + \frac{\partial}{\partial k_m^{ab}} I^2_{ab}(k_m^{ab}, k)\Big|_{k_m^{ab}=k_m} (k_m^{ab} - k_m)$$

Using Eq. 6-3 :

$$\tilde{I}_{ab}^{2} = \left(1+(E-k_m)\frac{\partial}{\partial k_m}\right) I_{ab}^{2}(k_m,k) - \frac{\partial}{\partial k_m}\left[(E_b-E_a) I^{2}(k_m,k)\right]\Big/_{k_m^{ab}=k_m} \qquad 6\text{-}6$$

Substituting Eq. 6-6 in Eq. 6-4, a new expression is obtained :

$$\tilde{I}_n(k_m,k) = \left(1+(E-k_m)\frac{\partial}{\partial k_m}\right) I_n(k_m,k) - \int_{-1}^{1} dy\, y^n \frac{\partial}{\partial k_m}\Big\{\sum_{ab} \frac{k(k_m^{ab}-k)^2}{m_\mu^3}$$

$$\times (E_b-E_a)\,\theta(k_m^{ab}-k)\, |\langle b|\sum_i J_-(i)\, e^{-i\vec{s}_{ab}\cdot\vec{r}_i}|a\rangle|^2 \Big/_{k_m^{ab}=k_m}\Big\} \qquad 6\text{-}7$$

where $I_n(k_m,k)$ is the closure approximation to $I_n(k)$. When substituted in Eq. 6-5 this yields an expression for the differential photon spectrum which will be denoted $\tilde{N}(k_m,k)$.

The final term in Eq. 6-7 can be calculated using a modified Thomas-Reiche-Kuhn sum rule (Er 70). For a many-particle Hamiltonian H = T+V, an operator $\sum_i O_i$ which satisfies the commutation relation $[\sum_i O_i, V] = 0$ then also satisfies :

$$\sum_b (E_b'-E_a')\, |\langle b|\sum_i O_i|a\rangle|^2 = \frac{1}{2m}\langle a|\sum_i (\vec{\nabla}_i O_i^{\dagger})\cdot(\vec{\nabla}_i O_i)|a\rangle \qquad 6\text{-}8$$

with E_a', E_b' eigenstates of H and m the nucleon mass.

There are two refinements which can now be made before applying this sum rule to the evaluation of Eq. 6-7. These have not generally been made in previous calculations for ordinary capture, but do seem to have numerical significance and so will be included here. The first deals with the Coulomb energy shift. Observe that Eq. 6-8 contains $E_b'-E_a'$, the difference in eigenvalues of the nuclear Hamiltonian,

whereas Eq. 6-6 involves $E_b - E_a$, the difference in the actual nuclear level energies, which of course includes the Coulomb energy. The two are related by $E_b - E_a = E'_b - E'_a - E_c$ where E_c is the Coulomb energy difference between the (A,Z) and (A,Z-1) ground states. Substitution of this relation into Eq. 6-7 effectively replaces E by $E+E_c$ in the second term.

The second refinement has to do with the influence of exchange corrections. When the TRK sum rule is applied to photoabsorption processes it fails, predicting total photoabsorption cross sections which are too low. This is not entirely unexpected since it has been known for some time that nuclear exchange potentials not satisfying $[\sum_i O_i, V_{exch}] = 0$ give rise to terms which make a significant contribution to the sum rule (Le 50, Be 57). The inclusion of exchange forces would enhance the right hand side of Eq. 6-8 by an amount conventionally described by a phenomenological factor $(1 + \delta)$, which for specific nuclei may be determined from fits to experimental total photoabsorption cross sections. Moreover since the processes of photoabsorption and muon capture are both dominated by dipole transitions and in that approximation involve similar operators, we may reasonably expect that values for δ determined from photoabsorption data (Ah 75) can be used to estimate the δ necessary to calculate muon capture rates. Thus use will be made of the modified TRK sum rule in the form :

$$\sum_b (E_b'-E_a') |\langle b| \sum_i O_i |a\rangle|^2 = \frac{(1+\delta)}{2m} \langle a| \sum_i (\vec{\nabla}_i O_i^{+}) \cdot (\vec{\nabla}_i O_i) |a\rangle \qquad 6\text{-}9$$

Applying Eq. 6-9 to the final term in Eq. 6-7, the differential photon spectrum obtained is :

$$\tilde{N}(k_m,k) = (\alpha\, m_\mu G^2/4\pi^2) |\varphi_\mu|^2 \sum_n C_n \tilde{I}_n(k_m,k) \qquad 6\text{-}10$$

with

$$\tilde{I}_n(k_m,k) = [1+(E+E_c-k_m)\frac{\partial}{\partial k_m}] I_n(k_m,k) - (1+\delta)\frac{2Zk}{m\, m_\mu^3} \times \begin{cases} (2k_m^3 - 6kk_m^2 + 7k^2k_m - 3k^3)/(n+1) & n \text{ even} \\ (3kk_m^2 - 6k^2k_m + 3k^3)/(n+2) & n \text{ odd} \end{cases} \qquad 6\text{-}11$$

To obtain the relative rate, the ordinary capture rate calculated in the same approximation is required. Thus the ordinary rate Λ is written (again neglecting velocity terms) as :

$$\Lambda = \frac{m_\mu^2}{2\pi} |\varphi_\mu|^2 \sum_{\bar{a}b} (G_F^2 + 3G_{GT}^2)_{ab} M_{ab}^2(\nu_{ab}) \qquad 6\text{-}12$$

with :

$$M_{ab}^2(\nu_{ab}) = (\nu_{ab}/m_\mu)^2 \int \frac{d\Omega_\nu}{4\pi} |\langle b| \sum_j \tau_-(j) e^{-i\vec{\nu}_{ab}\cdot\vec{r}_j} |a\rangle|^2 \qquad 6\text{-}13$$

Proceeding as for radiative capture the corrected rate is found to be :

$$\tilde{\Lambda}(\nu) = \frac{m_\mu^2}{2\pi} |\varphi_\mu|^2 (G_F^2 + 3G_{GT}^2) \tilde{M}^2(\nu) \qquad 6\text{-}14$$

where :

$$\tilde{M}^2(\nu) = [1+(E+E_c-\nu)\frac{d}{d\nu}] M^2(\nu) - 2(1+\delta) Z \frac{m_\mu}{m} \left(\frac{\nu}{m_\mu}\right)^3 \qquad 6\text{-}15$$

with $M^2(\nu)$ the usual closure result. This is essentially the result of Bernabeu (Be 73A), with Coulomb and exchange corrections added as above.

So far in this and previous work the velocity terms have been neglected. This has been necessary for the sum rule piece because an operator $\sim\vec{P}$ does not satisfy the assumptions necessary to obtain the simple sum rule of Eq. 6-9. Observe however that both the velocity terms and the sum rule corrections to the main terms are $O(1/m)$. Thus presumedly a sum rule correction to the velocity terms, which would involve commutators with the kinetic energy $\vec{P}^2/2m$, would be of $O(1/m^2)$, i.e. of the same order as other terms neglected. Corrections of $O(1/m)$ are obtained however by including the velocity terms in the usual closure results $\sum_n C_n I_n(k_m,k)$ and $(G_F^2 + 3G_{GT}^2)\ M^2(\nu)$ appearing in the first two terms of Eqs. 6-11 and 6-15. So henceforth the velocity terms will be incorporated in $\sum_n C_n I_n(k_m,k)$ and $(G_F^2 + 3G_{GT}^2)$ $\times M^2(\nu)$, yielding a result which is expected to be accurate to $O(1/m)$.

C. Sum Rule Relations for Average Excitation Energies

Before actually evaluating the expressions derived above in the context of a specific model, a discussion of the meaning of the parameters k_m and ν is presented. In the process, a consistency relation is derived by using the same sum rule techniques employed above. The relation allows us to make an explicit estimate of the "physical" values of k_m and ν.

So far k_m and ν have been somewhat loosely referred to as "average" maximum photon energy and "average" neutrino energy corresponding to an average nuclear excitation of the residual nucleus. Strictly speaking they are not averages in the physical sense but simply parameters defined formally by $N(k) = N(k_m,k)$ and $\Lambda = \Lambda(\nu)$ where $N(k_m,k)$ and $\Lambda(\nu)$ are the closure approximations to the actual rates $N(k)$ and Λ. Thus k_m and ν are the values which, when used to evaluate the rates in closure approximation, give the correct answer.

In the previous section improved approximations to the correct rates were obtained, i.e. $\tilde{N}(k_m,k)$ and $\tilde{\Lambda}(\nu)$. It will be shown in the following discussion that at least in a simple model these are relatively independent of k_m and ν. Thus the appropriate values of k_m and ν to use in a closure calculation can be estimated by solving the equations $\tilde{N}(k_m,k) = N(k_m,k)$ and $\tilde{\Lambda}(\nu) = \Lambda(\nu)$, that is, by determining the intersection point of the closure and corrected calculations. This is just the point where the two correction terms in Eq. 6-11 or those in Eq. 6-15 exactly cancel. The resulting parameters are then those which must be used in a closure approximation calculation for consistent results. They are of course somewhat dependent on the model used for the nuclear matrix elements and thus if drastically different values are required to fit the data, one should view the model with suspicion.

The "physical" values of average maximum photon energy

and average neutrino energy (which we shall write as $\bar{k}_m$ and $\bar{\nu}$) are in principle different from the parameters k_m and ν. Their p^{th} moments can be defined formally as :

$$\bar{k}_m^{\,p} = \sum_{\bar{a}b} (k_m^{ab})^p N_{ab}(k) \Big/ \sum_{\bar{a}b} N_{ab}(k)$$

$$\bar{\nu}^{\,p} = \sum_{\bar{a}b} (\nu_{ab})^p \Lambda_{ab} \Big/ \sum_{\bar{a}b} \Lambda_{ab} \qquad \text{6-16}$$

where the denominators are just the rates N(k) and Λ, and the numerators are the appropriate quantities for the transition $a \rightarrow b$ weighted by the rates for that transition and summed over all states b.

In closure approximation $\bar{k}_m = k_m$ and $\bar{\nu} = \nu$, which is what is normally assumed. Corrections to these relations can however be calculated in exactly the same way that corrections to the closure approximation for the rates were calculated, so that when such corrections are included, these equalities will no longer hold. To do this let us expand both numerator and denominator of Eq. 6-16 about $k_m^{ab} = k_m$ and $\nu_{ab} = \nu$ and carry through the sum rule evaluation as done in Eqs. 6-6 through 6-11. The results, keeping only the first order correction, are :

$$\bar{k}_m^{\,p} = k_m^p + p\,k_m^{p-1} \Big\{ (E + E_c - k_m) - \frac{(1+\delta)(Zk/m m_\mu^3)(k_m - k)^2}{\sum_n C_n I_n(k_m, k)}$$

$$\times \sum_n C_n \left\{ \begin{array}{ll} (k_m^2 - 2k(k_m - k))/(n+1) & n \text{ even} \\ 2k(k_m - k)/(n+2) & n \text{ odd} \end{array} \right\} \Big\} \qquad \text{6-17}$$

and :

$$\bar{\nu}^{p} = \nu^{p} + p\nu^{p-1}\{(E+E_c-\nu)-(1+\delta)\nu^2 Z/(2mM^2(\nu))\} \qquad 6\text{-}18$$

Thus for p=1 these relations express the physical averages $\bar{k}_m$ and $\bar{\nu}$ as the closure parameters k_m and ν plus a correction term. Note that the correction is essentially the same, only without the derivatives, as that appearing in the equations for $\tilde{N}(k_m, k)$ and $\tilde{\Lambda}(\nu)$.

A more useful result can be obtained by expanding the right sides of Eq. 6-16 about $\bar{k}_m$ and $\bar{\nu}$ instead of k_m and ν. The resulting equations (identical to those above with $k_m \rightarrow \bar{k}_m$ and $\nu \rightarrow \bar{\nu}$ everywhere) now provide consistency relations which can be solved for $\bar{k}_m$ and $\bar{\nu}$. Since the leading terms cancel, the solutions correspond to the zero of the term in brackets and provide an estimate of the physical averages corresponding to the physical average excitation energy of the residual nucleus. Note that the results will depend on the model used for the nuclear matrix elements in $I_n(k_m, k)$ and $M^2(\nu)$. These consistency relations will be evaluated for a simple model in the next section.

D. Application to ^{40}Ca

To illustrate the ideas outlined in the previous sections, an application of our method will be made to the nucleus ^{40}Ca, the nuclear matrix elements being evaluated using harmonic oscillator shell model wave functions. Such a model is perhaps too simple, but has been used for most comparisons with data and in any case will illustrate the basic results. In this model the closure result for I_n is :

$$I_n(k_m,k) = \frac{k(k_m-k)^2}{m_\mu^3}\,\theta(k_m-k)\int_{-1}^{1} dy\, y^n\, \mathcal{M}^2 \qquad 6\text{-}19$$

where for ^{40}Ca with $\eta^2 = (sb)^2$ the main nuclear matrix element is :

$$\mathcal{M}^2 = 20\left[1-\left(1+\frac{\eta^4}{8}-\frac{\eta^6}{80}+\frac{\eta^8}{640}\right)\exp(-\eta^2/2)\right] \qquad 6\text{-}20$$

The oscillator parameter is taken to be b = 2.03 fm. Expressions 6-5 and 6-10 for the differential photon spectrum were evaluated using the C_n from the appendix in Fearing (Fe 66) and the set of weak couplings $g_V = 1.0$, $g_M = 3.7$, $g_A = -1.25$, $g_P = 7\ g_A$, $g_S = 0$, $g_T = 0$. From Nolen and Schiffer (No 69) $E_c = 7.13$ Mev, and from Engfer et al (En 74) $E_{BE} = 1.066$ Mev, which give the value $E + E_c = 110.4$ Mev. As noted above, the velocity terms have been included as has the phenomenological correction for exchange effects, $(1+\delta)$.

It has been customary to present results for radiative muon capture rates as a ratio of the differential photon spectrum to the ordinary rate, as presumedly some of the model dependence of the nuclear matrix elements will then cancel, though factors which independently affect the overall scale of the amplitudes for radiative and for ordinary capture will of course also affect the ratio. In figures 29 and 30 are shown such plots which illustrate the main features of our results. It can be seen that the ratio of corrected quantities $\tilde{N}(k_m,k)/\tilde{\Lambda}(\nu)$, shown as solid and short-dashed curves for two different values of δ, is

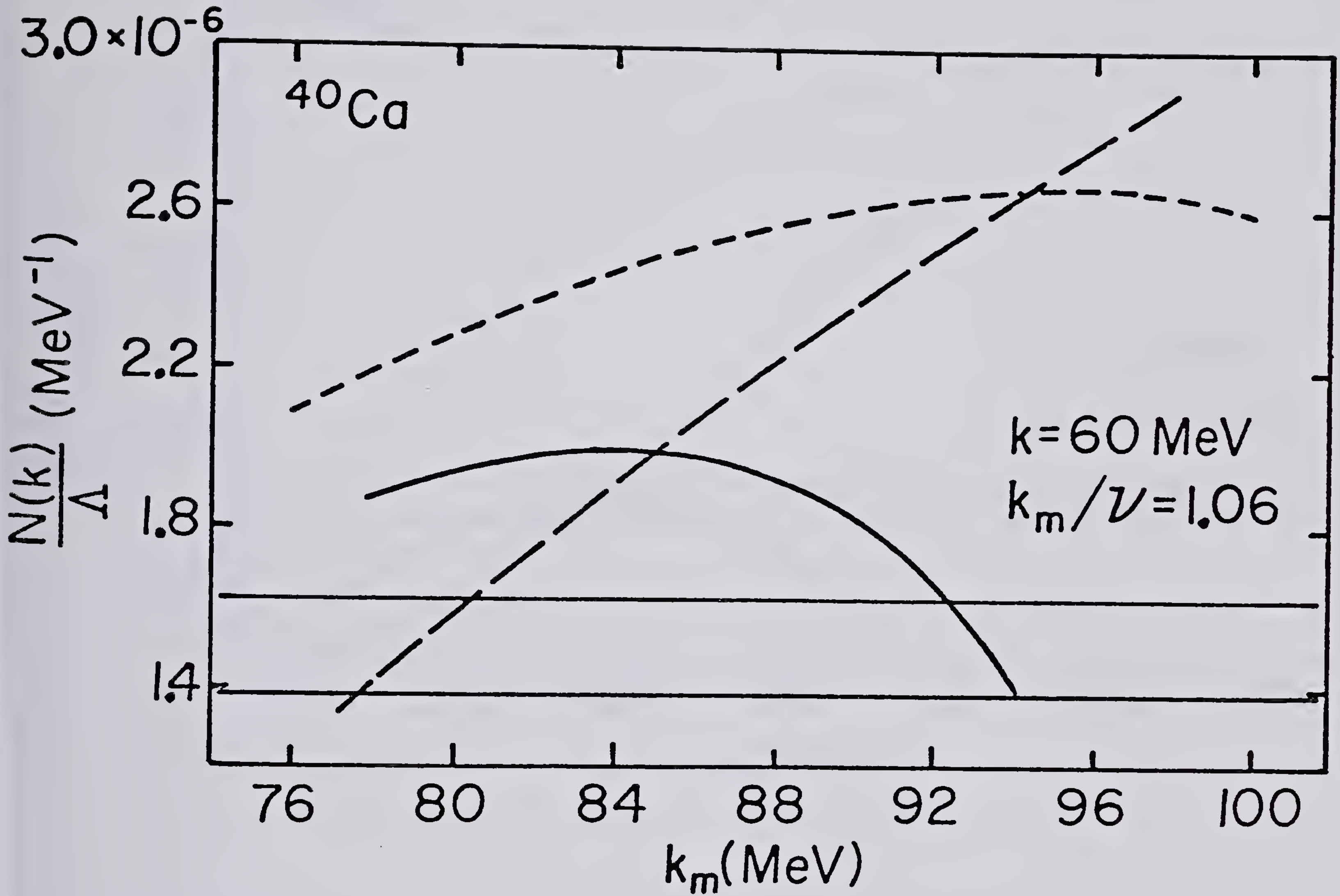

Fig. 29. The relative capture rate $N(k)/\Lambda$ for ^{40}Ca for $k=60$ Mev and $k_m/\nu=1.06$. The usual closure result (long-dashed curve) is compared to the corrected result for $\delta=0$ (short-dashed curve) and for $\delta=1.15$ (solid curve). The horizontal lines are experimental bounds on $N(k)/\Lambda$ from (Ha 77).

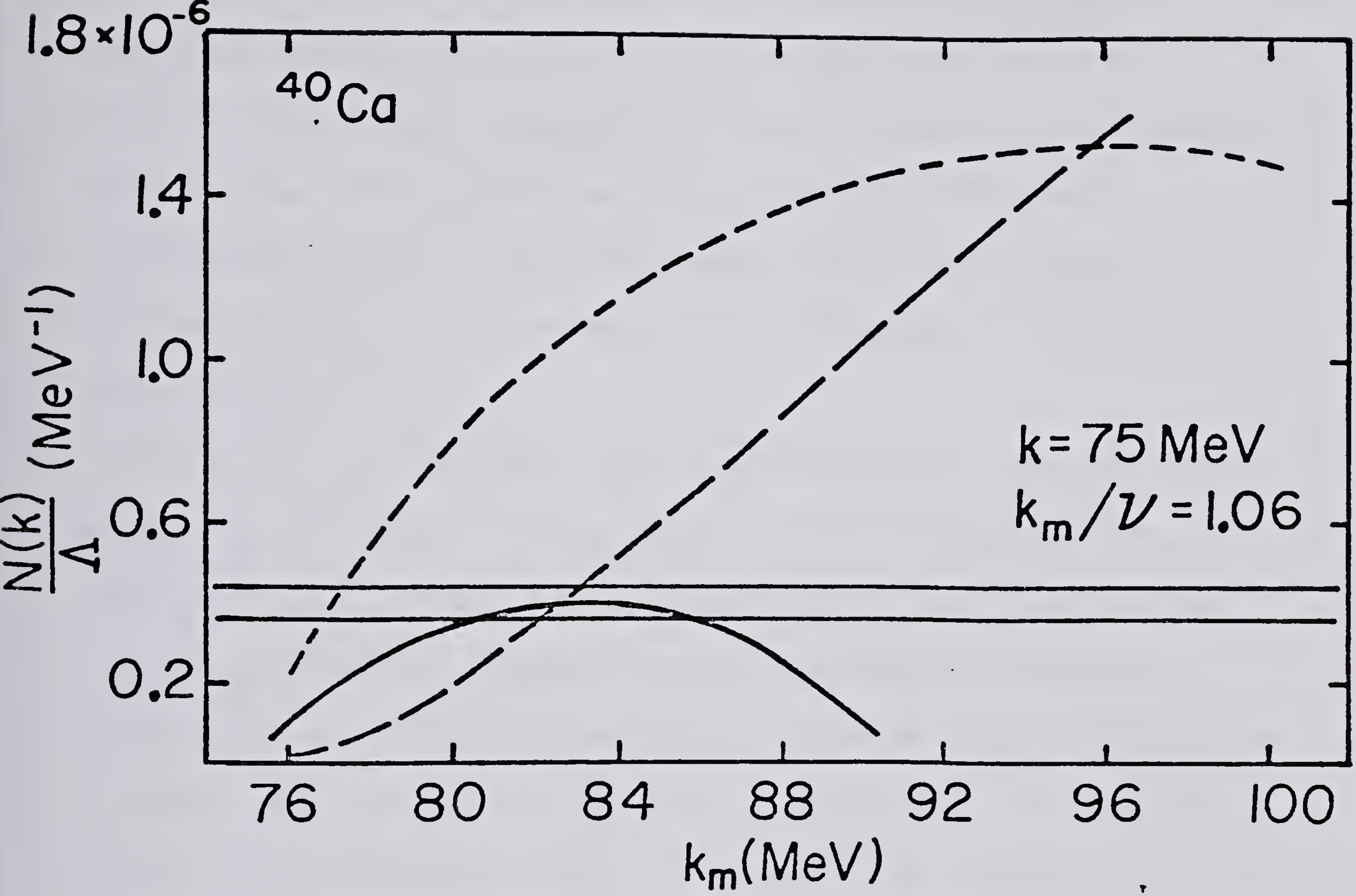

Fig. 30. The relative capture rate $N(k)/\Lambda$ for ^{40}Ca for k=75 Mev and k_m/ν=1.06. The curves have the same meaning as in figure 29.

generally much less dependent on k_m than the usual closure result $N(k_m,k)/\Lambda(\nu)$, shown as a long-dashed curve. Thus the important qualitative result is that for radiative capture, just as for ordinary capture (Be 73A), the sum rule technique allows one to obtain a result more or less independent of the closure parameter k_m over a reasonable range of k_m. Note that since $\tilde{\Lambda}(\nu)$ is itself nearly independent of ν in this approach, these qualitative features hold for the absolute rate $\tilde{N}(k_m,k)$ just as for the ratio $\tilde{N}(k_m,k)/\tilde{\Lambda}(\nu)$.

It is also observed that the curves for the corrected ratio $\tilde{N}(k_m,k)/\tilde{\Lambda}(\nu)$ and the closure ratio $N(k_m,k)/\Lambda(\nu)$ intersect in a region where $\tilde{N}(k_m,k)/\Lambda(\nu)$, and due to the stability of $\tilde{\Lambda}(\nu)$, $\tilde{N}(k_m,k)$ itself, is stable. Thus the intersection of $\tilde{N}(k_m,k)$ and $N(k_m,k)$ can be made a criterion in the selection of a value of k_m to be used in an ordinary closure calculation of $N(k)$. Similarly the appropriate value of ν can be determined from the intersection of $\tilde{\Lambda}(\nu)$ and $\Lambda(\nu)$. As emphasized in Sec. C, k_m and ν determined this way are basically just parameters which force the closure approximation to give results for $N(k_m,k)$ and $\Lambda(\nu)$ equal to the sum rule corrected values, which in turn approximate the correct results.

In figures 29 to 31 are shown results for two different values of δ, $\delta=0$ and $\delta=1.15$. The value $\delta=0$ corresponds to no exchange contribution correction to the sum rule piece, whereas $\delta=1.15$ is the lower limit on δ given by Ahrens et al

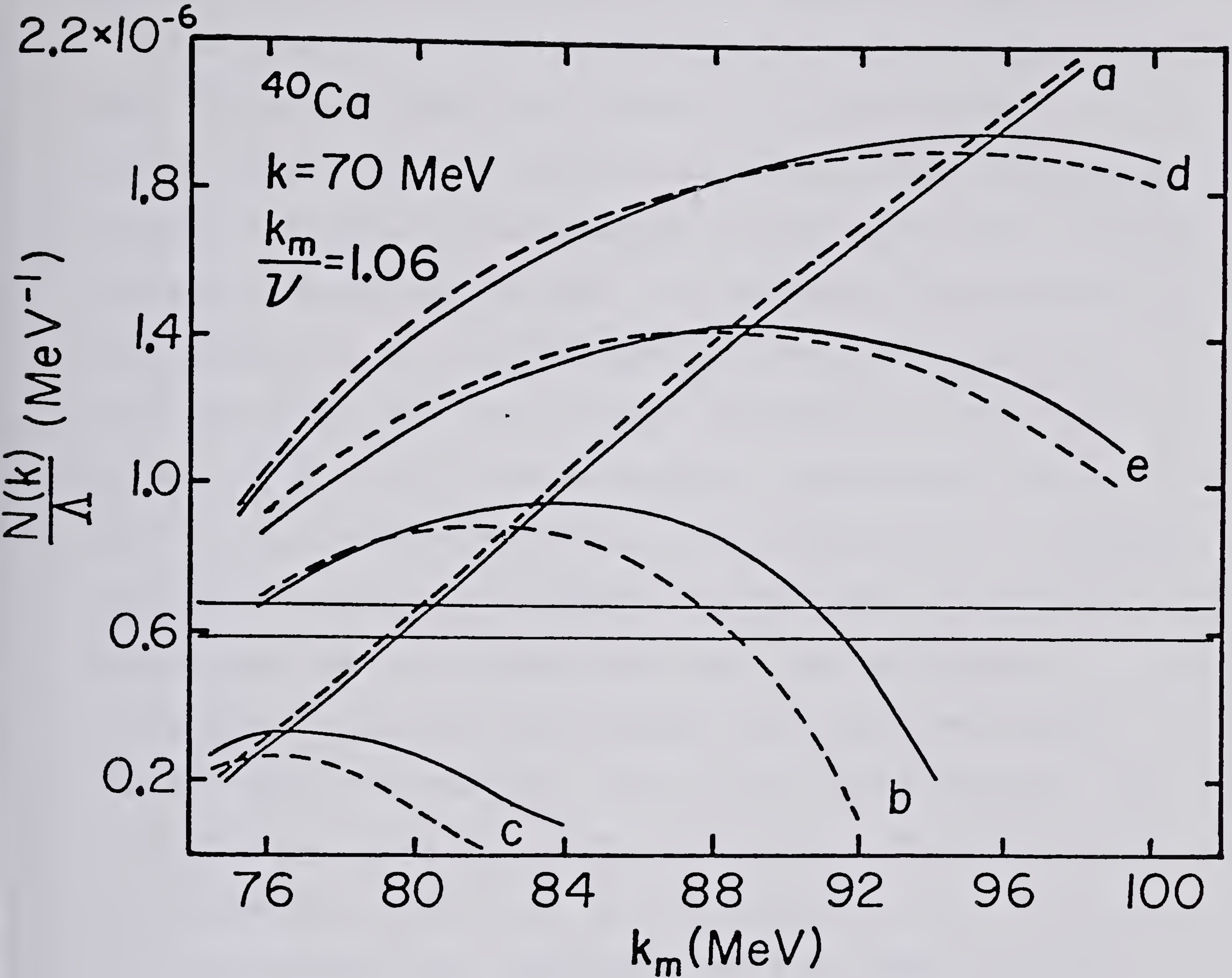

Fig. 31. Influence of the Coulomb energy shift and velocity terms on the ^{40}Ca relative capture rate for k=70 Mev and k_m/ν=1.06. Results including velocity terms (solid curves) and not including them (dashed curves) are shown for two choices of the phenomenological factor δ and with and without the Coulomb energy shift E_c. The horizontal lines are experimental bounds on N(k)/Λ from (Ha 77)
a - usual closure result
b - δ=1.15, E_c=7.1 Mev
c - δ=1.15, E_c=0
d - δ=0, E_c=7.1 Mev
e - δ=0, E_c=0.

(Ah 75) as determined from the ratio of the experimental photoabsorption cross section to the classical dipole sum. It is observed that the introduction of the correction δ is numerically important, particularly for large k. The correction is also important in principle since a consistent treatment of both photoabsorption and radiative muon capture using sum rules requires that the information available from photoabsorption be used to constrain the parameters for radiative muon capture. It seems reasonable to take the values for δ, $1.15 \leq \delta \leq 1.4$, directly from Ahrens et al (Ah 75), since the operators in the nuclear matrix elements for photoabsorption are essentially the same as for muon capture (Be 73C,Fo 64) and since for ^{40}Ca the dipole transition dominates both ordinary and radiative capture. For the minimum correction $\delta = 1.15$ the absolute rates are in much better agreement with the data (Ha 77) than for no correction (cf. Table II). As δ is increased from 1.15 to 1.4, the relative rate decreases, in most cases improving the agreement with the data. The current experimental uncertainty in δ corresponds to an uncertainty in k_m, determined from the intersection criterion, of about 1.5 Mev.

Figure 31 shows in addition the sensitivity of the results to the two other corrections which have been included, Coulomb energy shifts and velocity terms. The Coulomb correction is very important. Its inclusion tends to increase $\tilde{N}(k_m,k)/\tilde{\Lambda}(\nu)$ and the radiative rate $\tilde{N}(k_m,k)$ by as

Table II. Ordinary and radiative muon capture rates Λ and N, and corresponding average parameters ν and k_m, obtained from the intersection of the closure and corrected rate expressions for ^{40}Ca in a simple harmonic oscillator model. Also presented are values of the physical averages $\overline{\nu}$ and $\overline{k_m}$ obtained by solving the consistency relations (Eqs. 6-17 and 6-18) for these quantities. The cases are :

I: δ =1.15, E_c=7.1 Mev, velocity terms included;
II: δ =1.15, E_c=7.1 Mev, no velocity terms;
III: δ =1.15, E_c =0.0 Mev, velocity terms included;
IV: δ=0.0, E_c=7.1 Mev, velocity terms included;
V: δ=0.0, E_c =7.1 Mev, no velocity terms.

Table II

Ordinary and Radiative Muon Capture Rates Obtained from the Intersection Criterion

	k (MEV)	ν (MEV)	$\overline{\nu}$ (MEV)	k_m (MEV)	$\overline{k}_m$ (MEV)	k_m/ν	$\overline{k}_m/\overline{\nu}$	$10^{-7}\Lambda$ (SEC^{-1})	N (SEC^{-1} MEV^{-1})	$10^6 N/\Lambda$ (MEV^{-1})
I	60	78.72	82.76	84.08	84.93	1.068	1.026	0.333	6.55	1.97
	65			83.92	84.42	1.066	1.020		4.96	1.49
	70			83.53	83.73	1.061	1.012		3.15	0.95
	75			82.75	82.81	1.051	1.000		1.32	0.40
II	60	77.12	80.78	82.22	83.30	1.066	1.031	0.283	5.24	1.85
	65			82.31	83.02	1.067	1.028		3.96	1.40
	70			82.17	82.57	1.065	1.022		2.48	0.88
	75			81.74	81.89	1.060	1.014		0.98	0.35
III	60	73.41	76.94	77.86	78.22	1.061	1.017	0.259	3.65	1.40
	65			77.44	77.54	1.055	1.008		2.24	0.86
	70			76.67	76.64	1.044	0.996		0.83	0.32
	75			75.80	75.51	1.033	0.981		0.01	0.00
IV	60	93.12	96.05	96.99	97.94	1.042	1.020	0.599	15.98	2.67
	65			97.04	97.81	1.042	1.018		13.96	2.33
	70			97.04	97.62	1.042	1.016		11.73	1.96
	75			96.99	97.38	1.042	1.014		9.23	1.54
V	60	92.31	95.12	95.91	97.00	1.039	1.020	0.540	13.87	2.57
	65			96.01	96.96	1.040	1.019		12.20	2.26
	70			96.14	96.87	1.041	1.018		10.33	1.91
	75			96.18	96.71	1.042	1.017		8.17	1.51

much as a factor of two or three for some k in extreme cases. It also increases $\tilde{\Lambda}(\nu)$, though by a smaller amount, and increases k_m and ν obtained from the intersection criterion by roughly the Coulomb energy shift. Thus in this approach it is clearly important to consider the Coulomb energy.

The velocity terms are somewhat less important, but still should be included. They increase the ordinary rate by 15-20%, the radiative rate by 30-35%, and the ratio by 5-15%. This is a somewhat larger effect than is found in the usual closure approximation (where the velocity terms make very little difference in the ratio $N(k_m,k)/\Lambda(\nu)$) both because the corrected rates $\tilde{N}(k_m,k)$ and $\tilde{\Lambda}(\nu)$ are more sensitive to the velocity terms than their closure counterparts and because these changes in the rates change the values of k_m and ν obtained from the intersection criterion by about 1 Mev (cf. Table II) which in turn produces a change in the contribution of the leading terms.

Finally the results obtained for k_m and ν from the intersection criterion and those for the physical averages $\overline{k_m}$ and $\overline{\nu}$ obtained by solving the consistency relations of Eqs. 6-17 and 6-18 merit some discussion. These results are given in Table II where ν, k_m, k_m/ν, the ordinary and radiative rates and their ratio evaluated at k_m and ν, and the values $\overline{\nu}$, $\overline{k_m}$ and $\overline{k_m/\nu}$ are shown for several combinations of the corrections discussed above and for several values of k.

The most striking result is that for fixed δ, the ratio k_m/V is essentially constant even though k_m and V vary individually by sizeable amounts and for some k the rates can vary by as much as factors of two. Thus it seems sensible to fit data by fixing k_m/V and treating say k_m as a parameter, as was done by Hart et al (Ha 77). For the standard Rood and Tolhoek model (Ro 65), using simple harmonic oscillator wave functions and δ=1.15, it is seen from Table II that k_m/V = 1.06 ±0.01 is an appropriate value. For the less desireable choice δ=0, k_m/V is still fairly constant, but a bit lower, about 1.04.

These values of k_m/V are somewhat larger than those used before. This can be understood by looking at the ratio of "physical" averages $\overline{k_m}/\overline{V}$ obtained by solving the consistency relations starting from the definitions of Eq. 6-16. The values are not quite as constant as was the case for k_m/V, but still vary by only ± 00.02. They are generally lower than k_m/V. Values of k_m/V used in previous fits to data have usually been obtained from calculations such as that of Rood and Tolhoek (Ro 65), who got 1.02 for ^{16}C by comparing the closure result with that obtained by summing over partial transitions. Such a calculation starts with the definitions of "physical" averages (Eq. 6-16) however, and so is really a calculation of $\overline{k_m}/\overline{V}$. Hence the agreement with our consistency result for $\overline{k_m}/\overline{V}$ is encouraging. The effect of the additional sum rule correction terms is to increase slightly the value of k_m/V appropriate for a closure

calculation.

It should be emphasized that the specific numbers, e.g. for k_m/ν, may depend on the model used, in this case the harmonic oscillator model, though the qualitative features it is presumed are general. In a more complete calculation a number of additional corrections should be included, in particular the propagator corrections of Rood, Yano, and Yano (Ro 74), the higher order terms suggested by Fearing (Fe 75), perhaps better wave functions, and perhaps the requirement of consistency with electromagnetic matrix elements as done in the GDR model (Fe 66, Fo 64).

It is interesting to note that with δ=1.15 the ordinary rate 3.33 x 10^6 sec^{-1} is in fair agreement with the experimental values (2.29 ± 0.06) x 10^6 sec^{-1} (Ha 77) and (2.53 ± 0.02) x 10^6 sec^{-1} (Di 71). The radiative rate is somewhat high, but presumedly will be reduced on the order of 20% by the RYY correction (Ro 74), so that it may be possible to achieve qualitative agreement here as well. Alternatively, an increase in δ (δ=1.15 is a lower limit) reduces ordinary, radiative, and relative rates.

Thus to summarize, it has been shown that the differential photon spectrum for radiative muon capture can be calculated in a way which renders it nearly independent of the value of the maximum photon energy, in a fashion analogous to that for ordinary capture. The procedure consists of expanding the expression for the photon spectrum to first order about k_m, then using closure to perform the

sum over final states. Evaluation of one of the resulting correction terms is done using a modified TRK sum rule, leading to an expression for the photon spectrum which exhibits only slight dependence on k_m. The intersection of this result with the usual closure result determines values of k_m and ν for use in closure approximation calculations. From a practical point of view the most useful result may be that in the standard Rood and Tolhoek model (Ro 65) the sum rule correction gives values for k_m/ν which are essentially constant, and slightly larger than those used previously.

VII.

SUMMARY AND CONCLUSIONS

In the present work the ^{40}Ca photon spectrum has been calculated to $O(1/m^2)$ and the photon asymmetry calculated to $O(1/m^3)$, using the impulse approximation with the standard theory set of Feynman diagrams shown in fig. 1. While some of the $O(1/m^2)$ terms previously considered (Ro 65) are not negligible, the collective magnitude of previously neglected $O(1/m^2)$ and $O(1/m^3)$ terms was found to be small, resulting in a change over the experimentally accessible range k>55 Mev of less than 5% for both the photon spectrum and asymmetry as compared with the standard theory. Introduction of the SEL Hartree-Fock wave function (Sh 73) derived from the Feshbach-Lomon potential increased the absolute spectrum about 10% for b=1.70 fm (b=1.70 fm gave the best agreement with the measured ^{40}Ca charge form factor for $q<1$ fm^{-1}), however the relative spectrum was reduced less than 3%. This bears out the insensitivity of the relative spectrum to the choice of nuclear wave functions.

Two improvements in the GDR model - a more consistent choice of k_m and ν, and the use of new ^{40}Ca photoabsorption data - resulted in our finding that the GDR model relative photon spectrum is comparable in magnitude to the CHO model spectrum within 4% for 55<k<85 Mev. This result further indicates that the relative spectrum is to a large extent independent of the nuclear wave function. The GDR model thus predicts absolute and relative capture rates which are too

high by up to a factor of two, however both ordinary and radiative capture rates are in substantially better agreement with experiment than CHO model absolute rates. The discrepancy which existed prior to 1977 between experimental and theoretical determinations of the ^{40}Ca photon asymmetry has largely been resolved by a recent SREL experiment (Ha 77). Our calculation of the CHO model asymmetry is in agreement with previous calculations (Ro 65, Ro 74) and with this latest measurement. The GDR model photon asymmetry for ^{40}Ca, a quantity not previously calculated, was found to be nearly identical to the CHO model asymmetry.

The modified closure calculation presented in Chapter VI indicates a possible means of fixing k_m by appealing to experimental photoabsorption cross sections. Though our calculation contains a number of approximations, the underlying ideas can be pursued further if one is interested in improved accuracy. An undoubtedly temporary limitation in this regard is imposed by contemporary uncertainties in measured photoabsorption cross sections used to determine the modifying factor $(1+\delta)$. Results obtained for the ordinary capture rate by modifying the classical dipole sum are encouraging, and focus attention on the need for considering exchange currents in both ordinary and radiative muon capture.

As previously mentioned in our discussion of motivation for the GDR model, nucleon-nucleon correlations play a major role in determining the position of the giant dipole

resonance, and are capable of producing substantial changes in RMC observables. The shell model nuclear matrix elements described in Appendix D can be decomposed into one and two-body parts. As it turns out, the value of a matrix element depends to a large extent on the partial cancellation between the one and two-body parts. In a theory without two-body correlations such as ours, the degree of cancellation is mostly determined by the normalization of the wave functions. The introduction of two-body correlations into the theory might possibly affect the cancellation, thus bringing about a change in the value of a nuclear matrix element. An investigation of two-body correlations in ordinary muon capture by McCarthy and Walker (Mc 75) has shown that the capture rate for ^{16}O is reduced by 30% when correlations are included.

Lastly, to mention a closely related matter, the possibility of nuclear renormalization of the hadronic weak form factors has only recently received theoretical attention, and experimental information on this topic is still fairly sparse (Mu 76). The approach that has been followed so far consists of attempting to retain the framework of the impulse approximation and to replace the free weak couplings with effective ones. Indications that the axial vector strength is quenched up to 25% for medium and heavy mass nuclei (Br 78, Os 79, To 79) have strong implications for muon capture. Our results suggest that for g_A maximally quenched, both ordinary and radiative rates are

substantially reduced. This change is in the right direction to improve agreement with experimentally measured rates.

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Appendix A

Free-Particle Propagators for Muon and Nucleon

For the muon, the momentum space representation of the relativistic free propagator :

$$\tilde{S}_0(\mu) = \frac{\not{\mu} - m_\mu}{\mu^2 - m_\mu^2 + i\epsilon} \qquad \text{A-1}$$

was used directly in the evaluation of the lepton covariants $\mathcal{L}_\alpha$.

In the case of the nucleon, a free propagator consistent with our non-relativistic Hamiltonian was required. Accordingly it should satisfy the coordinate space equation :

$$\left(i\frac{\partial}{\partial t} - m - \frac{\vec{P}^2}{2m}\right) S_0(x-y) = \delta^4(x-y) \qquad \text{A-2}$$

where the operator $\left(i\frac{\partial}{\partial t} - m - \frac{\vec{P}^2}{2m}\right)$ is a Foldy-Wouthuysen reduction to $O(1/m^2)$ of the relativistic Hamiltonian for a free particle. Since the initial and final states of the nucleon involved in the RMC process are energy eigenstates, and the terms comprising the effective interaction Hamiltonian have position and time dependence :

$$H_E(x) \sim \exp(ik\cdot x)$$

$$H_W(x) \sim \exp(i(\nu-\mu)\cdot x) \qquad \text{A-3}$$

our problem is reduced to obtaining a suitable time-independent propagator for the nucleon which

satisfies :

$$\left(\frac{\vec{P}^2}{2m}+T\right)G_0(\vec{x}-\vec{y}) = \delta(\vec{x}-\vec{y}) \qquad \text{A-4}$$

with T=E-m, E being the total (relativistic) energy of the nucleon in the intermediate state. Thus if for example the neutron emits the photon, $T=T_N=E_f+k-m$, whereas if the proton radiates, $T=T_P=E_i-k-m$. Recognizing that Eq. A-4 is exactly the equation satisfied by the usual non-relativistic propagator for the free Schrodinger equation, one can immediately write the solution :

$$G_0(\vec{x}-\vec{y}) = \frac{1}{T}\int \frac{d\vec{P}}{(2\pi)^3}\,\frac{\exp(i\vec{P}\cdot(\vec{x}-\vec{y}))}{1-\vec{P}^2/2mT+i\epsilon} \qquad \text{A-5}$$

Further if k is large and $\vec{p}^2/2mT$ is small compared to 1, the integrand in Eq. A-5 can be expanded, yielding :

$$G_0(\vec{x}-\vec{y}) \simeq \frac{1}{T}\left\{1-\frac{\vec{\nabla}^2}{2mT}+\frac{\vec{\nabla}^4}{(2mT)^2}-\dots\right\}\delta(\vec{x}-\vec{y}) \qquad \text{A-6}$$

Considering the neutron radiating diagram specifically, the second order S-matrix takes the form :

$$S_{fi}^{(2)} \simeq \int d^3y\, d^3z\, \bar{\varphi}_f(\vec{y})\, H_E(\vec{y})\left\{\frac{1}{T_N}\left[1-\frac{\vec{\nabla}_y^2}{2mT_N}+\frac{\vec{\nabla}_y^4}{(2mT_N)^2}-\dots\right]\right.$$
$$\left.\times\,\delta(\vec{y}-\vec{z})\right\} H_W(\vec{z})\,\varphi_i(\vec{z}) \qquad \text{A-7}$$

Performing an integration by parts and making use of Eqs. A-3 yields :

$$S_{fi}^{(2)} \simeq \int d^3y\, \bar{\varphi}_f(\vec{y})\, H_E(\vec{y})\, H_W(\vec{y})\left\{\frac{1}{T_N}\left[1+\frac{(\vec{P}-\vec{\nu}-\vec{\mu})^2}{2mT_N}\right.\right.$$
$$\left.\left.+\frac{(\vec{P}-\vec{\nu}-\vec{\mu})^4}{(2mT_N)^2}+\dots\right]\right\}\varphi_i(\vec{y}) \qquad \text{A-8}$$

Similarly one obtains for the proton radiating diagram :

$$S_{fi}^{(2)} \simeq \int d^3y\, \bar{\varphi}_f(\vec{y})\, H_W(\vec{y})\, H_E(\vec{y}) \left\{ \frac{1}{T_P} \left[1 + \frac{(\vec{P}-\vec{k})^2}{2mT_P} + \frac{(\vec{P}-\vec{k})^4}{(2mT_P)^2} + \ldots \right] \right\} \varphi_i(\vec{y}) \qquad \text{A-9}$$

Note that the correct contact limit for the propagator (enclosed in braces in Eqs. A-8 and A-9) is obtained by letting the nucleon mass go to infinity :

$$\lim_{m\to\infty} G_0 = \begin{cases} +\frac{1}{k} & \text{neutron radiates} \\ -\frac{1}{k} & \text{proton radiates} \end{cases} \qquad \text{A-10}$$

At this point the initial nucleon was put on mass shell, an approximation consistent with the omission of external nuclear fields :

$$E_i - m = \frac{\vec{P}_i^{\,2}}{2m} \qquad \text{A-11}$$

Expanding T_P and T_N in the variable $\vec{p}^2/2mk$ and keeping terms formally to $O(1/m)$, the free propagator is approximated :

$$G_0 \sim \begin{cases} \frac{1}{(m_\mu-\nu)} + \frac{1}{2m}\frac{\nu^2}{(m_\mu-\nu)^2} - \frac{\nu}{m}\frac{\hat{\nu}\cdot\vec{P}}{m(m_\mu-\nu)^2} & \text{neutron radiates} \\ -\frac{1}{k} + \frac{1}{m} - \frac{\hat{k}\cdot\vec{P}}{mk} & \text{proton radiates} \end{cases} \qquad \text{A-12}$$

As it turns out, higher order terms in the expansion of the propagator beyond the first $\vec{P}/m$ term are not required if the squared matrix element is kept to $O(1/m^2)$; in fact keeping terms in the squared matrix element to second order meant that in most cases only the leading contact term in the expansion of the propagator was actually used.

Appendix B

RMC Effective Hamiltonian

Our effective Hamiltonian has the overall structure :

$$H^{eff} = \chi_\nu^{+} (1 - \vec{\sigma}_L \cdot \hat{\nu}) \mathcal{M} \chi_\mu \qquad \text{B-1}$$

with :

$$\mathcal{M} = \mathcal{M}_1 + \vec{\sigma}_L \cdot \vec{\mathcal{M}}_2 \qquad \text{B-2}$$

To facilitate manipulation by the algebraic squaring routines, $\mathcal{M}$ was arranged into the form :

$$\mathcal{M} = \sum_{j=1}^{Z} \mathcal{I}(j) \left\{ \sum_{i=1}^{N} (E_i + D_i + C_i) F_i + \sum_{i=N+1}^{M} (E_i + D_i + C_i) \vec{\sigma}_L \cdot \vec{G}_i \right\}_j e^{-i\vec{S}\cdot\vec{r}_j} \qquad \text{B-3}$$

where F_i and G_i are functions of the vectors $\vec{\epsilon}$, $\vec{k}$, $\vec{\nu}$, $\vec{P}/m$, and $\vec{\sigma}$. The coefficients E_i, D_i, and C_i correspond to terms of order 1, 1/m, and $1/m^2$ respectively. Table III lists the terms appearing in $\mathcal{M}_1$, while table IV lists the terms in $\vec{\mathcal{M}}_2$. A "Y" appended to a coefficient name denoted that the factor y= $\hat{k}\cdot\hat{\nu}$ appeared in the coefficient. These "Y" coefficients were segregated because an integration on y had to be done after the algebraic squaring. A complete list of the coefficients, as used by the numerical program NEWRMC, appears below. The FORTRAN symbols are explained in table V.

Table III

Terms in $\mathcal{M}_1$

i	F_i	Non-Zero Coefficients		
1	$i\hat{\epsilon}\cdot\hat{\nu}\times\vec{\sigma}$		D1	C1
2	$i\hat{\epsilon}\cdot\vec{P}\times\vec{\sigma}/m$			C2
3	$i\hat{\epsilon}\cdot\vec{P}\times\hat{\nu}/m$			C3
4	$\hat{\epsilon}\cdot\vec{\sigma}$	E4	D4	C4,C4Y
5	$\hat{\epsilon}\cdot\vec{P}/m$		D5	C5
6	$\hat{\epsilon}\cdot\hat{\nu}$			C6
7	$\hat{k}\cdot\vec{P}\ \hat{\epsilon}\cdot\vec{\sigma}/m$			C7
8	$\vec{P}\cdot\vec{P}\ \hat{\epsilon}\cdot\vec{\sigma}/m^2$			C8
9	$\vec{P}\cdot\hat{\nu}\ \hat{\epsilon}\cdot\vec{\sigma}/m$			C9
10	$\hat{\nu}\cdot\vec{\sigma}\ \hat{\epsilon}\cdot\hat{\nu}$		D10,D10Y	C10,C10Y
11	$\hat{k}\cdot\vec{\sigma}\ \hat{\epsilon}\cdot\hat{\nu}$		D11,D11Y	C11,C11Y
13	$\vec{P}\cdot\vec{\sigma}\ \hat{\epsilon}\cdot\hat{\nu}/m$			C13,C13Y
14	$\vec{P}\cdot\vec{\sigma}\ \vec{P}\cdot\hat{\epsilon}/m^2$			C14
15	$\vec{P}\cdot\hat{\epsilon}\ \hat{\nu}\cdot\vec{\sigma}/m$			C15
16	$\vec{P}\cdot\hat{\epsilon}\ \hat{k}\cdot\vec{\sigma}/m$			C16
17	$\vec{P}\cdot\hat{\epsilon}\ \vec{P}\cdot\hat{k}/m^2$			C17

Table III. Combinations of vectors appearing in the expression
for $\mathcal{M}_1$, with the effective Hamiltonian given by

$$H^{eff} = \chi_\nu^\dagger(1-\vec{\sigma}_L\cdot\hat{\nu})(\mathcal{M}_1+\vec{\sigma}_L\cdot\vec{\mathcal{M}}_2)\chi_\mu$$

The non-vanishing terms of order 1, 1/m, and $1/m^2$ are denoted by the E, D, and C coefficients respectively.

Table IV

Terms in $\vec{m}_2$

i	$\vec{G}_i$	Non-Zero Coefficients		
20	$i\hat{\epsilon}\times\vec{\sigma}$	E20	D20	C20,C20Y
21	$i\hat{\epsilon}\times\hat{\nu}$			C21
22	$i\hat{k}\cdot\vec{P}\ \hat{\epsilon}\times\vec{\sigma}/m$			C22
23	$i\vec{P}\cdot\vec{P}\ \hat{\epsilon}\times\vec{\sigma}/m^2$			C23
24	$i\hat{\nu}\cdot\vec{P}\ \hat{\epsilon}\times\vec{\sigma}/m$			C24
25	$i\hat{\epsilon}\ \hat{\nu}\cdot\vec{\sigma}\times\vec{P}/m$			C25
26	$i\hat{\epsilon}\ \hat{k}\cdot\vec{\sigma}\times\vec{P}/m$			C26
27	$i\hat{k}\ \hat{\nu}\cdot\vec{\sigma}\times\hat{\epsilon}$			C27
28	$i\hat{k}\ \hat{\epsilon}\cdot\vec{\sigma}\times\vec{P}/m$			C28
29	$i\hat{\epsilon}\cdot\hat{\nu}\ \vec{P}\times\vec{\sigma}/m$			C29
30	$i\hat{\epsilon}\cdot\hat{\nu}\ \hat{\nu}\times\vec{\sigma}$			C30
31	$i\hat{\epsilon}\cdot\hat{\nu}\ \hat{k}\times\vec{\sigma}$			C31
32	$i\vec{P}\cdot\vec{\sigma}\ \hat{\epsilon}\times\hat{\nu}/m$			C32
33	$i\vec{P}\cdot\vec{\sigma}\ \vec{P}\times\hat{\epsilon}/m^2$			C33
34	$i\hat{\nu}\cdot\vec{\sigma}\ \vec{P}\times\hat{\epsilon}/m$			C34
35	$i\hat{k}\cdot\vec{\sigma}\ \vec{P}\times\hat{\epsilon}/m$			C35
37	$i\hat{\epsilon}\times\vec{P}/m$		D37	C37
38	$i\vec{P}\cdot\hat{\epsilon}\ \hat{\nu}\times\vec{\sigma}/m$			C38
39	$\hat{\epsilon}$	E39	D39	C39,C39Y

Table IV. Combinations of vectors appearing in the expression for $\vec{m}_2$, with the effective Hamiltonian given by

$$H^{eH} = \chi_\nu^\dagger (1-\vec{\sigma}_L\cdot\hat{\nu})(m_1+\vec{\sigma}_L\cdot\vec{m}_2)\chi_\mu$$

The non-vanishing terms of order 1, 1/m, and 1/m² are denoted by the E, D, and C coefficients respectively.

Table IV (continued)

Terms in $\vec{\mathcal{M}}_2$

i	$\vec{G}_i$	Non-Zero Coefficients	
40	$\hat{\epsilon}\ \vec{P}\cdot\vec{\sigma}/m$	D40	C40,C40Y
41	$\hat{k}\ \hat{\epsilon}\cdot\vec{\sigma}$	D41	C41
42	$\vec{\sigma}\ \hat{\epsilon}\cdot\hat{\nu}$	D42	C42
43	$\hat{\epsilon}\ \vec{\sigma}\cdot\hat{\nu}$	D43,D43Y	C43,C43Y
44	$\vec{P}\ \hat{\epsilon}\cdot\vec{\sigma}/m$		C44
45	$\hat{\epsilon}\ \vec{P}\cdot\hat{k}/m$		C45
46	$\hat{\epsilon}\ \vec{P}\cdot\vec{P}/m^2$		C46
47	$\hat{\epsilon}\ \vec{P}\cdot\hat{\nu}/m$		C47
48	$\vec{\sigma}\ \hat{\epsilon}\cdot\vec{P}/m$	D48	C48
50	$\hat{k}\ \hat{\epsilon}\cdot\vec{P}/m$		C50
51	$\vec{P}\ \hat{\epsilon}\cdot\hat{\nu}/m$		C51
52	$\vec{P}\ \hat{\epsilon}\cdot\vec{P}/m^2$		C52
53	$\hat{\epsilon}\ \vec{\sigma}\cdot\hat{k}$	D53,D53Y	C53,C53Y

Table V

FORTRAN Symbol Dictionary

FORTRAN Symbol	Variable
L	λ
LPL	λ_+
LM	λ_-
MMU	m_μ
M	m
MPI	m_π
MUP	χ_P
MUN	χ_N
MUA	$\chi_P + \chi_N$
MUS	$\chi_P - \chi_N$
MU	$1 + \chi_P - \chi_N$
KS	$\lvert\vec{k}\rvert$
K	$\hat{k}$
NUS	$\lvert\vec{\nu}\rvert$
NU	$\hat{\nu}$
FSDK	$\hat{\xi}\cdot\hat{k}$ term flag
FVEL	velocity term flag
FMR	muon radiating diagram term flag
KM	k_m
FCP	contact propagator term flag
GM	g_M
GP	g_P

Table V. Dictionary of FORTRAN symbols appearing in the computer listings of the squared matrix element.

Table V (continued)

FORTRAN Symbol Dictionary

FORTRAN Symbol	Variable
FEP	expanded propagator term flag
ET	$m_\mu/2m$
KP	$k_m/2m$
X	k/k_m
XNUS	ν/k_m
AL	$\alpha = (m_\pi^2 - m_\mu^2 + 2m_\mu k_m)/(m_\pi^2 + m_\mu^2)$
BE	$\beta = 2m_\mu k_m/(m_\pi^2 + m_\mu^2)$
GAM	$\gamma = \alpha^{-1}$
DEL	$\delta = 2k_m^2\gamma^2/(m_\pi^2 + m_\mu^2)$
GPN	$g_P^N = g_P/(\alpha - \beta X)$
GPL	$g_P^L \equiv g_P[\gamma + \delta X(1-X)(1-y)]$
RPO	$-k^{-1}$
RP1	$(2m)^{-1}$
RNO	$(m_\mu - \nu)^{-1}$
RN1	$(\nu^2/2m)/(m_\mu - \nu)^2$
FNM	=0 (unused flag)
appended G	denotes complex conjugation
GS	g_S
GT	g_T
GV	g_V
GA	g_A

[illegible]

E Coefficients

```
E4=-GA*LPL*FMR
E20=GA*LPL*FMR
E39=GS*(1.-LPL*FMR)-GV*LPL*FMR
```

D Coefficients

```
 D1=-(GV+GM)*LPL*KP*XNUS*FMR
 D4=(-GPN-GA+GT)*ET
&  +(GV+GM)*LPL*KP*(1.-2.*X)*FMR
&  +GT*LPL*(KP-ET)*FMR
&  -(GS+GV)*L*KP*X*MMU*((1.+MUP)*RPO+MUN*RNO)
 D5=-GV*LPL*FMR-(GS+GV)*MMU*RPO
 D10=GPN*BE*KP*XNUS**2*(GAM+DEL*X*XNUS)
 D10Y=-GPN*BE*KP*XNUS**2*(DEL*X*XNUS)
 D11=GPN*BE*KP*X*XNUS*(GAM+DEL*X*XNUS)
 D11Y=-GPN*BE*KP*X*XNUS*(DEL*X*XNUS)
 D20=-(GV+GM)*ET
&  -GT*LPL*(KP-ET)*FMR
&  -GA*L*KP*X*MMU*((1.+MUP)*RPO-MUN*RNO)
 D37=GV*LPL*FMR
 D39=-GV*ET
&  -GV*LPL*KP*FMR
&  -GA*L*KP*X*MMU*((1.+MUP)*RPO+MUN*RNO)
 D40=-GA*LPL*FMR
 D41=-(GV+GM)*LPL*KP*X*FMR
 D42=(GV+GM)*LPL*KP*XNUS*FMR
 D43=GPN*(AL-BE*X)*(GAM+DEL*X*XNUS)*KP*XNUS*(1.-LPL*FMR)
&  +(GA+GT)*LPL*KP*XNUS*FMR
 D43Y=-GPN*(AL-BE*X)*(DEL*X*XNUS)*KP*XNUS*(1.-LPL*FMR)
 D48=-GA*MMU*RPO
 D53=GPN*(AL-BE*X)*(GAM+DEL*X*XNUS)*KP*X*(1.-LPL*FMR)
&  +(GA+GT)*LPL*KP*X*FMR
 D53Y=-GPN*(AL-BE*X)*(DEL*X*XNUS)*KP*X*(1.-LPL*FMR)
```

C Coefficients

```
 C1=GPN*L*KP*ET*XNUS*KS*((1.+MUP)*RP0-MUN*RN0)
 C1=C1+GS*ET*KP*XNUS-2.*GS*ET*KP*XNUS*KS*MUN*RN0
 C1=C1-GV*ET*KP*XNUS-2.*GV*ET*KP*XNUS*KS*MUN*RN0
& -GV/2.*LPL*KP*XNUS*(KP-ET)*FMR
 C1=C1-GA*ET*KP*XNUS*L*KS*((1.+MUP)*RP0-MUN*RN0)
 C1=C1-2.*GM*ET*KP*XNUS
& -GM*LPL*KP*XNUS*(KP-ET)*FMR
 C1=C1-GT*L*ET*KP*XNUS*KS*((1.+MUP)*RP0-MUN*RN0)
& -GT*LPL*KP**2*XNUS**2*FMR
 C2=-GS*ET/2.+GS/2.*ET*KS*((1.+2.*MUP)*RP0+2.*MUN*RN0)
 C2=C2+GV/2.*ET+GV/2.*ET*KS*((1.+2.*MUP)*RP0+2.*MUN*RN0)
& +GV/2.*LPL*(KP-ET)*FMR
 C2=C2+GA*ET*L*KS*((1.+MUP)*RP0-MUN*RN0)
 C2=C2+GM*ET+GM*LPL*(KP-ET)*FMR
 C2=C2+GT*LPL*KP*XNUS*FMR
 C3=GA/2.*LPL*KP*XNUS*FMR
 C4=GPN/2.*ET*(-ET+KP*(XNUS-X*(1.+2.*MUA)))
& -(GS+GV)*ET*L*KS*((1.+MUP)*RP1+MUN*RN1)
& -GA*LPL*FMR*KP**2*(XNUS**2+X**2)*FNM
 C4=C4+GS*L*ET*KP*X*1.5
& -GS/2.*L*ET*KP*X*KS*((1.+2.*MUP)*RP0+2.*MUN*RN0)
 C4=C4-GV/2.*L*ET*KP*X
& -GV/2.*L*ET*KP*X*KS*((1.+2.*MUP)*RP0+2.*MUN*RN0)
& -GV*L*ET*KP*XNUS*KS*((1.+MUP)*RP0+MUN*RN0)
& +GV/2.*LPL*(KP-ET)*KP*(XNUS-X)*FMR
 C4=C4-GA/2.*ET*(KP*XNUS+ET-KP*(1.+2.*MUS))
& +2.*GA*ET*KP*XNUS*KS*MUN*RN0
& -2.*GA*ET*KP*X*KS*(1.+MUP)*RP0
& +GA/2.*LPL*(X**2+XNUS**2)*KP**2*FMR
 C4=C4-GM*L*ET*KP*X+GM*LPL*(KP-ET)*KP*(XNUS-X)*FMR
 C4=C4+GT*LPL*KP**2*(XNUS**2+X**2-X*XNUS)*FMR
& +GT*ET*KP*XNUS
 C4Y=-GA*LPL*FMR*KP**2*2.*X*XNUS*FNM
 C4Y=C4Y+GA/2.*LPL*(2.*X*XNUS)*KP**2*FMR
 C4Y=C4Y+GT*LPL*KP**2*(2.*X*XNUS)*FMR
 C5=-(GS+GV)*MMU*RP1
```

```
      C5=C5+GS*ET
      C5=C5-GV*KP*XNUS*RPO*MMU
      C5=C5-GA*ET*L*KS*((1.+MUP)*RPO+MUN*RNO)
     & +GA/2.*LPL*KP*(XNUS-X)*FMR
      C6=-GPN*L*ET*KP*XNUS*KS*((1.+MUP)*RPO+MUN*RNO)
      C6=C6-GS*ET*KP*XNUS
      C6=C6+GV*ET*KP*XNUS
     & +GV/2.*LPL*(KP-ET)*KP*XNUS*FMR
      C6=C6+GA*L*ET*KP*XNUS*KS*((1.+MUP)*RPO+MUN*RNO)
      C6=C6+2.*GM*ET*KP*XNUS
     & +GM*LPL*KP*XNUS*(KP-ET)*FMR
      C6=C6+GT*L*ET*KP*XNUS*KS*((1.+MUP)*RPO+MUN*RNO)
      C7=(GS+GV)*ET*(1.+MUP)*L*FEP
     & -GA*LPL*KP*X*FMR*FNM
      C7=C7-GA*LPL*KP*X*FMR
      C7=C7-GT*LPL*KP*X*FMR
      C8=-GA*LPL/2.*FMR*FNM
      C8=C8-GA/2.*LPL*FMR
      C9=(GS+GV)*ET*MUN*L*KP**2*X*XNUS/(ET-KP*XNUS)**2*FEP
     & -GA*LPL*KP*XNUS*FMR*FNM
      C9=C9-GA*LPL*KP*XNUS*FMR
      C9=C9-GT*LPL*KP*XNUS*FMR
      C10=GPN*KP*BE*XNUS**2*(GAM+DEL*X*XNUS)/2.*(KP-ET)
      C10=C10-GT*LPL*KP**2*XNUS**2*FMR
      C10Y=-GPN*KP*BE*XNUS**2*(DEL*X*XNUS)/2.*(KP-ET)
      C11=GPN*KP*BE*X*XNUS*(GAM+DEL*X*XNUS)/2.*(KP-ET)
      C11=C11-GT*LPL*KP**2*X*XNUS*FMR
      C11Y=-GPN*KP*BE*X*XNUS*(DEL*X*XNUS)/2.*(KP-ET)
      C13=-GPN*KP*BE*XNUS*(1.-ET/KP)*(GAM+DEL*X*XNUS)/2.
      C13=C13+GA/2.*LPL*KP*XNUS*FMR
      C13Y=GPN*KP*BE*XNUS*(1.-ET/KP)*(DEL*X*XNUS)/2.
      C14=-GA*RPO*MMU-GA/2.*LPL*FMR
      C15=-GPN*ET*KP*XNUS*2.*RPO
      C15=C15+GA*KP*XNUS*MMU*RPO
     & +GA/2.*LPL*KP*XNUS*FMR
      C15=C15+GT*KP*XNUS*MMU*RPO
```

```
&  +GT*LPL*KP*XNUS*FMR
 C16=GA/2.*LPL*KP*X*FMR+2.*GA*ET*KS*RP0
 C17=(GS+GV+GA)*ET/(KP*X)*FEP
 C20=-GA*ET*L*KS*((1.+MUP)*RP1-MUN*RN1)
&  +GA*LPL*KP**2*(X**2+XNUS**2)*FMR*FNM
 C20=C20+GV/2.*ET*(KP*XNUS-ET-KP*X*(1.+2.*MUA))
 C20=C20-GA/2.*ET*L*KP*X*KS*((1.+2.*MUP)*RP0-2.*MUN*RN0)
&  -GA/2.*LPL*KP**2*(X**2+XNUS**2)
 C20=C20+GM*ET*(KP*XNUS-ET)
 C20=C20+GT*L*KP*X*(KP*XNUS-ET)*((1.+MUP)*RP0-MUN*RN0)
 C20Y=GA*LPL*KP**2*(2.*X*XNUS)*FMR*FNM
 C20Y=C20Y-GA/2.*LPL*(2.*X*XNUS)*KP**2
 C21=GV*L*ET*KP*XNUS*KS*((1.+MUP)*RP0+MUN*RN0)
&  -GV/2.*LPL*(KP-ET)*KP*XNUS*FMR
 C21=C21+GA*ET*KP*XNUS-2.*GA*ET*KP*XNUS*KS*MUN*RN0
 C21=C21+GM*L*ET*KP*XNUS*KS*((1.+MUP)*RP0+MUN*RN0)
&  -GM*LPL*KP*XNUS*(KP-ET)*FMR
 C22=GA*ET*L*(1.+MUP)*FEP
&  +GA*LPL*KP*X*FMR*FNM
 C22=C22+GA*LPL*KP*X*FMR
 C23=GA*LPL/2.*FMR*FNM
 C23=C23-GA/2.*LPL*FMR
 C24=-GA*ET*MUN*L*X*KP**2*XNUS/(ET-KP*XNUS)**2
&  +GA*LPL*KP*XNUS*FMR*FNM
 C24=C24+GA*KP*XNUS*FMR*LPL
 C25=-GS/2.*KP*XNUS*(1.-LPL*FMR)
 C25=C25-GV/2.*LPL*KP*XNUS*FMR
 C25=C25-GM*LPL*KP*XNUS*FMR
 C26=-GS/2.*KP*X*(1.-LPL*FMR)
 C26=C26-GV/2.*LPL*KP*X*FMR
 C26=C26-GM*LPL*KP*X*FMR
 C27=GT*LPL*KP**2*X*XNUS*FMR
 C28=GT*LPL*KP*X*FMR
 C29=GT*LPL*KP*XNUS*FMR
 C30=-GT*LPL*KP**2*XNUS**2*FMR
 C31=-GT*LPL*KP**2*X*XNUS*FMR
```

```
 C32=-GA/2.*LPL*KP*XNUS*FMR
 C33=-GA/2.*LPL*FMR
 C34=GA/2.*LPL*KP*XNUS*FMR
 C35=GA/2.*LPL*KP*X*FMR
 C37=-GA/2.*ET+GA/2.*ET*KS*((1.+2.*MUP)*RP0+2.*MUN*RN0)
 C38=-GV*KP*XNUS*MMU*RP0
 C38=C38-GM*KP*XNUS*RP0
 C39=-GA*ET*L*KS*((1.+MUP)*RP1+MUN*RN1)
& +GS*(1.-LPL*FMR)*KP**2*(X**2+XNUS**2)*FNM
 C39=C39-GS/2.*KP**2*(X**2+XNUS**2)*(1.-LPL*FMR)
 C39=C39+GV/2.*ET*(KP*XNUS-ET-KP*X*(1.+2.*MUA))
& -GV/2.*LPL*(KP-ET)*KP*X*FMR
& +GV/2.*LPL*KP**2*(X**2+XNUS**2)*FMR
 C39=C39+GA*1.5*L*ET*KP*X
& -GA/2.*L*ET*KP*X*KS*((1.+2.*MUP)*RP0+2.*MUN*RN0)
 C39=C39+GM*ET*(KP*XNUS-ET)
& +GM*LPL*KP*(KP*XNUS*(XNUS-X)+X*ET)*FMR
 C39=C39+GT*L*ET*(KP*XNUS-ET)*KS*((1.+MUP)*RP0+MUN*RN0)
& -GT*L*ET*KP*X
 C39Y=GS*(1.-LPL*FMR)*KP**2*(2.*X*XNUS)*FNM
 C39Y=C39Y-GS/2.*KP**2*(2.*X*XNUS)*(1.-LPL*FMR)
 C39Y=C39Y+GV/2.*LPL*KP**2*(2.*X*XNUS)*FMR
 C39Y=C39Y+GM*LPL*KP**2*(2.*XNUS*X)*FMR
 C40=-GPN*(AL-BE*X)*(GAM+DEL*X*XNUS)/2.*(KP-ET)*(1.-LPL*FMR)
 C40=C40-GA/2.*ET+GA/2.*ET*KS*((1.+2.*MUP)*RP0-2.*MUN*RN0)
& -GA/2.*LPL*KP*X*FMR
 C40Y=GPN*(AL-BE*X)*(DEL*X*XNUS)/2.*(KP-ET)*(1.-LPL*FMR)
 C41=2.*GV*ET*KP*X*L*KS*(1.+MUP)*RP0
& -GV/2.*LPL*KP*X*(KP-ET)*FMR
 C41=C41+GA/2.*ET*KP*X
 C41=C41-GM*LPL*KP*X*(KP-ET)*FMR
 C41=C41-GT*ET*KP*X+GT*LPL*KP**2*X**2*FMR
 C42=-GV*ET*KP*XNUS*L*KS*((1.+MUP)*RP0-MUN*RN0)
& +GV/2.*LPL*KP*XNUS*(KP-ET)*FMR
 C42=C42-GA*KP*XNUS*ET
 C42=C42-GM*L*ET*KP*XNUS*KS*((1.+MUP)*RP0-MUN*RN0)
```

```
&  +GM*LPL*KP*XNUS*(KP-FT)*FMR
  C43=GPN*(AL-BE*X)*(GAM+DEL*X*XNUS)/2.*KP*XNUS*(KP-FT)*(1.-LPL*FMR)
  C43=C43+GV*L*ET*KP*XNUS*KS*((1.+MUP)*RP0-MUN*RN0)
  C43=C43+GA*ET*KP*XNUS+2.*GA*FT*KP*XNUS*KS*MUN*RN0
&  +GA/2.*LPL*KP*XNUS*(KP-ET)*FMR
  C43=C43+GM*L*KP*XNUS*ET*KS*((1.+MUP)*RP0-MUN*RN0)
  C43=C43+GT*ET*KP*XNUS
  C43Y=-GPN*(AL-BE*X)*(DEL*X*XNUS)/2.*KP*XNUS*(KP-ET)*(1.-LPL*FMR)
  C44=-GV*L*ET*KS*((1.+MUP)*RP0+MUN*RN0)
&  +GV/2.*LPL*(KP-ET)*FMR
  C44=C44-GA/2.*FT-GA/2.*ET*KS*((1.+2.*MUP)*RP0-2.*MUN*RN0)
  C44=C44+GM*LPL*(KP-ET)*FMR
  C44=C44+GT*ET
  C45=GA*ET*L*(1.+MUP)*FEP
&  +GS*(1.-LPL*FMR)*KP*X*FNM
  C45=C45+GS*KP*X*(1.-LPL*FMR)
  C46=GS/2.*(1.-LPL*FMR)*FNM
  C46=C46-GS/2.*(1.-LPL*FMR)
  C47=GA*ET*L*MUN*FEP*KP**2*X*XNUS/(FT-KP*XNUS)**2
&  +GS*(1.-LPL*FMR)*KP*XNUS*FNM
  C47=C47+GS*KP*XNUS*(1.-LPL*FMR)
  C48=-GV/2.*LPL*(KP-ET)*FMR
  C48=C48+GA*ET-GA*MMU*RP1
  C48=C48-GM*LPL*(KP-ET)*FMR
  C48=C48+GT*(KP*XNUS-ET)*MMU*RP0-GT*ET
  C50=2.*GV*FT*KS*RP0
  C50=C50-GA/2.*LPL*KP*X*FMR
  C51=GA/2.*LPL*KP*XNUS*FMR
  C52=-GV*RP0
  C53=GPN*(AL-BE*X)*(GAM+DEL*X*XNUS)/2.*KP*X*(KP-ET)*(1.-LPL*FMR)
  C53=C53+GA/2.*LPL*KP*X*(KP-ET)*FMR+GA*1.5*FT*KP*X
  C53Y=-GPN*(AL-BE*X)*(DEL*X*XNUS)/2.*KP*X*(KP-ET)*(1.-LPL*FMR)
```

Appendix C

RMC Matrix Element Squared

The REDUCE2 programs SQUARE1, SQUARE2, and SQUARE3 performed the algebraic squaring of $\mathcal{M}$, with output from SQUARE3 being done in the FORTRAN language. The complete expression for was not squared all at once, rather it was divided into segments according to powers of 1/m (via the E, D, and C coefficients of Appendix B), and the indivdual segments processed. Listed below are the parts of $\mathcal{M}^2$ which were calculated :

1. MS01 : $(O(1))^2 + O(1) \times O(1/m)$
2. MS012: $(O(1)+O(1/m))^2$
3. MS2 : $O(1) \times O(1/m^2)$
4. MS3 : $O(1/m) \times O(1/m^2)$

The output files MS01, MS012, MS2, and MS3 are reproduced below. Products of nuclear matrix elements are denoted :

$$PnYm \equiv \int_{-1}^{1} dy\, y^m \times POLn \qquad \text{C-1}$$

where the expressions POLn are defined in Appendix D.

MS01

```
C     I AND I/M TERMS
C                          TERM1
       FTOUT=
      XFTOUT-4*L*W*P1Y2*D1*E4+4*L*P1Y1*D1*E4+2*W*P1Y3*E4*D10-4*W*D4*
      XP1Y1*E4-2*W*P1Y1*E4**2-2*W*P1Y1*E4*D10+2*W*P1Y4*E4*D10Y-2*W*P1Y2*
      XE4*D10Y-2*P1Y3*E4*D10Y+4*D4*E4*P1Y0+2*P1Y1*E4*D10Y-2*P1Y2*E4*D10+
      X2*E4**2*P1Y0+2*E4*D10*P1Y0
C                          TERM2
       FTOUT=
      XFTOUT-(4*L*W*E20*E4*P1Y0-2*L*W*E20*P1Y3*D10Y+2*L*W*E20*D10Y*
      XP1Y1-2*L*W*E20*D10*P1Y2+2*L*W*E20*D10*P1Y0+4*L*W*E20*P1Y0*D4+4*
      XL*W*E4*D20*P1Y0-2*L*D11Y*E20*P1Y3+2*L*D11Y*E20*P1Y1-2*L*D11*E20
      X*P1Y2+2*L*D11*E20*P1Y0-4*L*E20*E4*P1Y1-4*L*E20*P1Y1*D4-4*L*E4*
      XP1Y1*D20+4*W*E20*D1*P1Y1-4*W*E4*D41*P1Y0+2*D48*E4*P2Y2*W2*XNUS*
      XKP-2*D48*E4*W2*P2Y0*XNUS*KP-2*D43*E4*P1Y2+2*D43*E4*P1Y0+2*XNUS*
      XE4*P2Y2*W2*D40*KP-2*XNUS*E4*W2*D40*P2Y0*KP+2*E39*P2Y2*W2*XNUS*KP
      X*D5-2*E39*W2*P2Y0*XNUS*KP*D5-2*E20*D1*P1Y2-2*E20*D1*P1Y0-2*E4*
      XD43Y*P1Y3+2*E4*D43Y*P1Y1-2*E4*P1Y2*D42+2*E4*D42*P1Y0+4*E4*P1Y1*
      XD41)
C                          TERM3
       FTOUT=
      XFTOUT+2*L*W*D43Y*P1Y3*E4-2*L*W*D43Y*P1Y1*E4+2*L*W*D43*P1Y2*E4-2*L
      X*W*D43*P1Y0*E4-2*L*W*P1Y2*D42*E4-2*L*W*P1Y2*D1*E20+2*L*W*P1Y0*D42
      X*E4+2*L*W*P1Y0*D1*E20-2*L*D5*XNUS*P2Y2*W3*KP*E39+2*L*D5*XNUS*P2Y0
      X*W3*KP*E39+2*L*XNUS*D48*P2Y2*W3*KP*E4-2*L*D48*XNUS*P2Y0*W3*KP*E4-2
      X*L*XNUS*P2Y2*W3*KP*D40*E4+2*L*XNUS*P2Y0*W3*KP*D40*E4+2*W*P1Y4*
      XD10Y*E20+2*W*P1Y3*D11Y*E20+2*W*P1Y3*D10*E20-2*W*P1Y2*D10Y*E20+2*W
      X*P1Y2*D11*E20-2*W*P1Y1*D11Y*E20-2*W*P1Y1*D10*E20-2*W*P1Y0*D11*E20
C                          TERM4
       FTOUT=
      XFTOUT+4*L*W*D43Y*P1Y3*E20+4*L*W*D53Y*P1Y2*E20+4*L*W*D43*P1Y2*E20-
      X4*L*W*D41*P1Y1*E20+4*L*W*D53*P1Y1*E20-4*L*X*P2Y0*W2*KP*D37*E39-4*
      XL*X*P2Y0*W2*KP*D40*E20-4*L*X*P2Y1*KP*W3*D37*E39-4*L*X*P2Y1*KP*W3*
      XD40*E20-4*L*W2*P2Y1*KP*D37*XNUS*E39-4*L*W2*P2Y1*KP*XNUS*D40*E20-4
      X*L*KP*W3*D37*P2Y2*XNUS*E39-4*L*KP*W3*P2Y2*XNUS*D40*E20+4*L*D43Y*
      XP1Y2*E20+4*L*D53Y*P1Y1*E20+4*L*D43*P1Y1*E20-4*L*D41*P1Y0*E20+4*L*
```

```
XD53*P1Y0*E20+4*W*D39*P1Y1*E39+8*W*D20*P1Y1*E20+2*W*P1Y1*E39**2+4*
XW*P1Y1*E20**2+4*D39*P1Y0*E39+8*D20*P1Y0*E20+2*P1Y0*E39**2+4*P1Y0*
XE20**2
C                          TERM5
  FTOUT=
XFTOUT-2*L*W*P1Y3*D43Y*E20+2*L*W*D43Y*P1Y1*E20-2*L*W*P1Y2*D43*E20
X+2*L*W*P1Y0*D43*E20+8*L*W*P1Y1*D41*E20-2*L*D40*P2Y0*KP*XNUS*W3*E20
X+2*L*D40*KP*XNUS*W3*P2Y2*E20-2*L*P2Y0*KP*XNUS*W3*D37*E39+2*L*KP
X*XNUS*W3*P2Y2*D37*E39-8*W*D20*P1Y1*E20-4*W*P1Y1*E20**2
C                          TERM6
  FTOUT=
XFTOUT+2*L*W*P1Y0*E39**2+4*L*W*P1Y0*E39*D39+2*L*W*P1Y0*E20**2+4*L*
XW*P1Y0*E20*D20+2*L*P1Y1*E39**2+4*L*P1Y1*E39*D39+2*L*P1Y1*E20**2+4
X*L*P1Y1*E20*D20+4*W*D53Y*P1Y1*E20+4*W*D53*P1Y0*E20+4*W*D43*P1Y1*
XE20+4*W*P1Y2*D43Y*E20+4*D53Y*P1Y2*E20+4*D53*P1Y1*E20+2*P1Y0*D43*
XE20-4*P1Y0*D42*E20+2*P1Y3*D43Y*E20+2*D43*P1Y2*E20+4*P1Y2*D42*E20+2
X*D43Y*P1Y1*E20-4*E39*W3*P2Y1*XNUS*D37*KP-4*X*P2Y0*E39*W3*D37*KP-2*
XE39*P2Y0*W2*XNUS*D37*KP-2*E39*W2*P2Y2*XNUS*D37*KP-4*X*P2Y1*E39*W2*
XD37*KP-4*E20*W3*P2Y1*XNUS*KP*D40-4*X*D40*E20*W3*P2Y0*KP-4*X*D40*
XE20*P2Y1*W2*KP+4*E20*P2Y0*W2*XNUS*KP*D48-2*E20*P2Y0*W2*XNUS*KP*D40
X-4*E20*W2*P2Y2*XNUS*KP*D48
   FTOUT=FTOUT-2*E20*W2*P2Y2*XNUS*KP*D40
```

MS012

```
C     SQUARE OF 1/M * 1/M
C                        TERM1
        FTOUT=
       XFTOUT+2*L*W*D1*D11Y*P1Y2-2*L*W*D1*D11Y*P1Y4+2*L*W*D1*P1Y1*D11-4*L
       X*W*D1*P1Y2*D4-2*L*W*D1*P1Y3*D11-2*L*D1*D11Y*P1Y1+2*L*D1*D11Y*P1Y3
       X+4*L*D1*P1Y1*D4+2*L*D1*P1Y2*D11-2*L*D1*P1Y0*D11-W*D1**2*P1Y1-W*D1
       X**2*P1Y3-W*D11Y**2*P1Y3+W*D11Y**2*P1Y5-2*W*D11Y*P1Y2*D11-2*W*D11Y
       X*P1Y3*D10-2*W*D11Y*P1Y4*D10Y+2*W*D11Y*P1Y4*D11+2*W*D11Y*P1Y5*D10+
       X2*W*D11Y*P1Y6*D10Y-W*P1Y1*D10**2-2*W*P1Y1*D10*D4-2*W*P1Y1*D4**2-W
       X*P1Y1*D11**2-2*W*P1Y2*D10*D10Y-2*W*P1Y2*D10*D11-2*W*P1Y2*D10Y*D4+
       XW*P1Y3*D10**2+2*W*P1Y3*D10*D4-W*P1Y3*D10Y**2-2*W*P1Y3*D10Y*D11+W*
       XP1Y3*D11**2+2*W*P1Y4*D10*D10Y+2*W*P1Y4*D10*D11+2*W*P1Y4*D10Y*D4+W
       X*P1Y5*D10Y**2
        FTOUT=FTOUT+2*W*P1Y5*D10Y*D11-X**2*W5*P5Y1*KP**2*D5**2+X**2*W4*KP
       X**2*D5**2*P5Y0-2*X*W5*P5Y2*XNUS*KP**2*D5**2+2*X*W4*P5Y1*XNUS*KP**
       X2*D5**2+D1**2*P1Y2+D1**2*P1Y0+D11Y**2*P1Y2-D11Y**2*P1Y4+2*D11Y*
       XP1Y1*D11+2*D11Y*P1Y2*D10+2*D11Y*P1Y3*D10Y-2*D11Y*P1Y3*D11-2*D11Y*
       XP1Y4*D10-2*D11Y*P1Y5*D10Y+2*P1Y1*D10*D10Y+2*P1Y1*D10*D11+2*P1Y1*
       XD10Y*D4-P1Y2*D10**2-2*P1Y2*D10*D4+P1Y2*D10Y**2+2*P1Y2*D10Y*D11-
       XP1Y2*D11**2-2*P1Y3*D10*D10Y-2*P1Y3*D10*D11-2*P1Y3*D10Y*D4-P1Y4*
       XD10Y**2-2*P1Y4*D10Y*D11-W5*P5Y3*XNUS**2*KP**2*D5**2+W5*P4Y1*D5**2
       X-W5*P3Y1*D5**2+W4*P5Y2*XNUS**2*KP**2*D5**2-W4*D5**2*P4Y0+W4*D5**2
       X*P3Y0
        FTOUT=FTOUT+D10**2*P1Y0+2*D10*D4*P1Y0+2*D4**2*P1Y0+P1Y0*D11**2
C                        TERM2
        FTOUT=
       XFTOUT+4*L*W*P1Y1*D41*D1-2*L*W*P1Y1*D10Y*D20+2*L*W*P1Y2*D10*D20+2*
       XL*W*P1Y2*D42*D1+2*L*W*P1Y3*D10Y*D20-4*L*W*D4*D20*P1Y0-2*L*W*D10*
       XD20*P1Y0-2*L*W*D42*D1*P1Y0-2*L*X**2*D37*D5*KP**2*W5*P5Y0+2*L*X**2
       X*D37*D5*KP**2*W4*P5Y1-4*L*X*D37*D5*XNUS*KP**2*W5*P5Y1+2*L*X*D37*
       XD5*XNUS*KP**2*W4*P5Y0+2*L*X*D37*D5*XNUS*KP**2*W4*P5Y2+2*L*X*KP*
       XP2Y2*W2*D40*D1-2*L*X*KP*W2*D40*D1*P2Y0-2*L*D37*D5*XNUS**2*KP**2*
       XW5*P5Y2+2*L*D37*D5*XNUS**2*KP**2*W4*P5Y1+2*L*D37*D5*W5*P4Y0-2*L*
       XD37*D5*W5*P3Y0-2*L*D37*D5*W4*P4Y1+2*L*D37*D5*W4*P3Y1+4*L*P1Y1*D4*
       XD20-2*L*P1Y1*D11Y*D20+2*L*P1Y1*D53Y*D1+2*L*P1Y2*D11*D20-2*L*P1Y2*
       XD53*D1
```

```
      FTOUT=FTOUT-4*L*P1Y2*D41*D1+2*L*P1Y3*D11Y*D20-2*L*P1Y3*D53Y*D1-2*L
     X*D11*D20*P1Y0+2*L*D53*D1*P1Y0+2*W*P1Y1*D10*D42+2*W*P1Y1*D41*D10Y+
     X2*W*P1Y1*D42*D11Y-4*W*P1Y1*D20*D1-2*W*P1Y2*D10*D41-2*W*P1Y2*D11*
     XD42+2*W*P1Y2*D42*D10Y-2*W*P1Y3*D10*D42-2*W*P1Y3*D41*D10Y-2*W*P1Y3
     X*D42*D11Y-2*W*P1Y4*D42*D10Y+4*W*D4*D41*P1Y0+2*W*D10*D41*P1Y0+2*W*
     XD11*D42*P1Y0+2*X*KP*P2Y1*W2*D10*D40+2*X*KP*P2Y1*W2*D11Y*D40-2*X*
     XKP*P2Y2*W2*D11*D40+2*X*KP*P2Y2*W2*D10Y*D40-2*X*KP*P2Y3*W2*D10*D40
     X-2*X*KP*P2Y3*W2*D11Y*D40-2*X*KP*P2Y4*W2*D10Y*D40+2*X*KP*W2*D11*
     XD40*P2Y0-2*D5*XNUS*KP*P2Y2*D39*W2+2*D5*XNUS*KP*D39*W2*P2Y0+2*XNUS
     X*KP*P2Y1*W2*D11*D40
      FTOUT=FTOUT+2*XNUS*KP*P2Y1*W2*D10Y*D40-2*XNUS*KP*P2Y2*W2*D4*D40-2*
     XXNUS*KP*P2Y2*W2*D10*D40+2*XNUS*KP*P2Y2*W2*D11Y*D40-2*XNUS*KP*P2Y3
     X*W2*D11*D40-2*XNUS*KP*P2Y3*W2*D10Y*D40-2*XNUS*KP*P2Y4*W2*D11Y*D40
     X+2*XNUS*KP*W2*D4*D40*P2Y0+2*XNUS*KP*W2*D10*D40*P2Y0-2*D43*P1Y1*
     XD11-2*D43*P1Y1*D10Y+2*D43*P1Y2*D4+2*D43*P1Y2*D10-2*D43*P1Y2*D11Y+
     X2*D43*P1Y3*D11+2*D43*P1Y3*D10Y+2*D43*P1Y4*D11Y-2*D43*D4*P1Y0-2*
     XD43*D10*P1Y0-2*D43Y*P1Y1*D4-2*D43Y*P1Y1*D10-2*D43Y*P1Y2*D11-2*
     XD43Y*P1Y2*D10Y+2*D43Y*P1Y3*D4+2*D43Y*P1Y3*D10-2*D43Y*P1Y3*D11Y+2*
     XD43Y*P1Y4*D11+2*D43Y*P1Y4*D10Y+2*D43Y*P1Y5*D11Y-4*P1Y1*D4*D41-2*
     XP1Y1*D10*D53
      FTOUT=FTOUT-2*P1Y1*D10*D41-2*P1Y1*D11*D42-2*P1Y1*D11*D53Y-2*P1Y1*
     XD53*D11Y-2*P1Y1*D42*D10Y+2*P1Y2*D4*D42+2*P1Y2*D10*D42-2*P1Y2*D10*
     XD53Y+2*P1Y2*D11*D53-2*P1Y2*D53*D10Y-2*P1Y2*D41*D10Y-2*P1Y2*D42*
     XD11Y-2*P1Y2*D11Y*D53Y+2*P1Y2*D20*D1+2*P1Y3*D10*D53+2*P1Y3*D10*D41
     X+2*P1Y3*D11*D42+2*P1Y3*D11*D53Y+2*P1Y3*D53*D11Y+2*P1Y3*D42*D10Y-2
     X*P1Y3*D10Y*D53Y+2*P1Y4*D10*D53Y+2*P1Y4*D53*D10Y+2*P1Y4*D41*D10Y+2
     X*P1Y4*D42*D11Y+2*P1Y4*D11Y*D53Y+2*P1Y5*D10Y*D53Y-2*D4*D42*P1Y0-2*
     XD10*D42*P1Y0-2*D11*D53*P1Y0+2*D20*D1*P1Y0
C                        TERM3
       FTOUT=
     XFTOUT-2*L*W*P1Y1*D11Y*D53-2*L*W*P1Y1*D53*D10-2*L*W*P1Y1*D43*D11-
     X2*L*W*P1Y1*D43*D10Y-2*L*W*P1Y1*D53Y*D11-2*L*W*P1Y1*D43Y*D4-2*L*W*
     XP1Y1*D43Y*D10-2*L*W*D11Y*P1Y2*D43-2*L*W*D11Y*P1Y2*D53Y+2*L*W*D11Y
     X*P1Y3*D53-2*L*W*D11Y*P1Y3*D43Y+2*L*W*D11Y*P1Y4*D43+2*L*W*D11Y*
     XP1Y4*D53Y+2*L*W*D11Y*P1Y5*D43Y-2*L*W*P1Y2*D20*D1-2*L*W*P1Y2*D42*
     XD4+2*L*W*P1Y2*D53*D11-2*L*W*P1Y2*D53*D10Y+2*L*W*P1Y2*D43*D4+2*L*W
```

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X*P1Y2*D43*D10-2*L*W*P1Y2*D53Y*D10-2*L*W*P1Y2*D43Y*D11-2*L*W*P1Y2*
XD43Y*D10Y+2*L*W*P1Y3*D53*D10+2*L*W*P1Y3*D43*D11+2*L*W*P1Y3*D43*
XD10Y+2*L*W*P1Y3*D53Y*D11-2*L*W*P1Y3*D53Y*D10Y+2*L*W*P1Y3*D43Y*D4+
X2*L*W*P1Y3*D43Y*D10
 FTOUT=FTOUT+2*L*W*P1Y4*D53*D10Y+2*L*W*P1Y4*D53Y*D10+2*L*W*P1Y4*
XD43Y*D11+2*L*W*P1Y4*D43Y*D10Y+2*L*W*P1Y5*D53Y*D10Y+2*L*W*D20*D1*
XP1Y0+2*L*W*D42*D4*P1Y0-2*L*W*D53*D11*P1Y0-2*L*W*D43*D4*P1Y0-2*L*W
X*D43*D10*P1Y0+2*L*X*D40*D11Y*W3*KP*P2Y1-2*L*X*D40*D11Y*W3*KP*P2Y3
X+2*L*X*D40*W3*KP*D10*P2Y1-2*L*X*D40*W3*KP*D10*P2Y3-2*L*X*D40*W3*
XKP*D11*P2Y2+2*L*X*D40*W3*KP*D11*P2Y0+2*L*X*D40*W3*KP*D10Y*P2Y2-2*
XL*X*D40*W3*KP*D10Y*P2Y4+2*L*D40*D11Y*W3*XNUS*KP*P2Y2-2*L*D40*D11Y
X*W3*XNUS*KP*P2Y4-2*L*D40*W3*XNUS*KP*D4*P2Y2+2*L*D40*W3*XNUS*KP*D4
X*P2Y0-2*L*D40*W3*XNUS*KP*D10*P2Y2+2*L*D40*W3*XNUS*KP*D10*P2Y0+2*L
X*D40*W3*XNUS*KP*D11*P2Y1
 FTOUT=FTOUT-2*L*D40*W3*XNUS*KP*D11*P2Y3+2*L*D40*W3*XNUS*KP*D10Y*
XP2Y1-2*L*D40*W3*XNUS*KP*D10Y*P2Y3-2*L*W3*XNUS*KP*D5*D39*P2Y2+2*L*
XW3*XNUS*KP*D5*D39*P2Y0-2*W*P1Y1*D11Y*D20-2*W*P1Y1*D20*D10+2*W*
XP1Y1*D1*D42+2*W*P1Y1*D1*D53Y+2*W*D11Y*P1Y3*D20+2*W*P1Y2*D20*D11-2
X*W*P1Y2*D20*D10Y-2*W*P1Y2*D1*D53+2*W*P1Y3*D20*D10-2*W*P1Y3*D1*D42
X-2*W*P1Y3*D1*D53Y+2*W*P1Y4*D20*D10Y-2*W*D20*D11*P1Y0+2*W*D1*D53*
XP1Y0+2*X*D40*D1*W3*KP*P2Y2-2*X*D40*D1*W3*KP*P2Y0-2*X*W5*XNUS*KP**
X2*D5*D37*P5Y2+2*X*W5*XNUS*KP**2*D5*D37*P5Y0+2*W5*XNUS**2*KP**2*D5
X*D37*P5Y1-2*W5*XNUS**2*KP**2*D5*D37*P5Y3
C                     TERM4
 FTOUT=
XFTOUT+4*L*W*P1Y1*D53*D20-4*L*W*P1Y1*D41*D20+4*L*W*P1Y2*D43*D20+4*
XL*W*P1Y2*D53Y*D20+4*L*W*P1Y3*D43Y*D20-4*L*X*D37*D39*W3*KP*P2Y1-4*
XL*X*D37*D39*W2*P2Y0*KP-4*L*X*W3*KP*P2Y1*D40*D20-4*L*X*W2*P2Y0*KP*
XD40*D20-4*L*D37*D39*W3*XNUS*KP*P2Y2-4*L*D37*D39*W2*XNUS*KP*P2Y1+4
X*L*P1Y1*D43*D20+4*L*P1Y1*D53Y*D20+4*L*P1Y2*D43Y*D20-4*L*W3*XNUS*
XKP*P2Y2*D40*D20-4*L*W2*XNUS*KP*P2Y1*D40*D20+4*L*P1Y0*D53*D20-4*L*
XP1Y0*D41*D20+2*W*D39**2*P1Y1+3*W*P1Y1*D42**2+2*W*P1Y1*D42*D43+2*W
X*P1Y1*D53**2+2*W*P1Y1*D41**2+2*W*P1Y1*D43**2+4*W*P1Y1*D20**2+2*W*
XP1Y2*D42*D43Y+4*W*P1Y2*D53*D43+4*W*P1Y2*D53*D53Y+4*W*P1Y2*D43*
XD43Y
 FTOUT=FTOUT-3*W*P1Y3*D42**2-2*W*P1Y3*D42*D43+4*W*P1Y3*D53*D43Y+4*W
```

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X*P1Y3*D43*D53Y+2*W*P1Y3*D53Y**2+2*W*P1Y3*D43Y**2-2*W*P1Y4*D42*
XD43Y+4*W*P1Y4*D53Y*D43Y-X**2*D37**2*W5*P5Y1*KP**2-X**2*D37**2*W4*
XP5Y0*KP**2-2*X*D37**2*W5*P5Y2*XNUS*KP**2-2*X*D37**2*W4*P5Y1*XNUS*
XKP**2-4*X*W3*KP*D53*P2Y1*D40-4*X*W3*KP*P2Y2*D43*D40-4*X*W3*KP*
XP2Y2*D53Y*D40-4*X*W3*KP*P2Y3*D43Y*D40-4*X*W2*P2Y0*KP*D53*D40-4*X*
XW2*KP*P2Y1*D43*D40-4*X*W2*KP*P2Y1*D53Y*D40-4*X*W2*KP*P2Y2*D43Y*
XD40-D37**2*W5*P5Y3*XNUS**2*KP**2+D37**2*W5*P4Y1+D37**2*W5*P3Y1-
XD37**2*W4*P5Y2*XNUS**2*KP**2+D37**2*W4*P4Y0+D37**2*W4*P3Y0+2*D39
X**2*P1Y0
  FTOUT=FTOUT+2*P1Y1*D42*D43Y+4*P1Y1*D53*D43+4*P1Y1*D53*D53Y+4*P1Y1*
XD43*D43Y-3*P1Y2*D42**2-2*P1Y2*D42*D43+4*P1Y2*D53*D43Y+4*P1Y2*D43*
XD53Y+2*P1Y2*D53Y**2+2*P1Y2*D43Y**2-2*P1Y3*D42*D43Y+4*P1Y3*D53Y*
XD43Y+2*W5*P3Y1*D40**2+2*W4*P3Y0*D40**2-2*W3*XNUS*KP*D42*P2Y1*D40+
X2*W3*XNUS*KP*D42*P2Y3*D40-4*W3*XNUS*KP*D53*P2Y2*D40-4*W3*XNUS*KP*
XP2Y1*D43*D40-4*W3*XNUS*KP*P2Y2*D43Y*D40-4*W3*XNUS*KP*P2Y3*D53Y*
XD40-2*W2*P2Y0*XNUS*KP*D42*D40-4*W2*P2Y0*XNUS*KP*D43*D40+2*W2*XNUS
X*KP*D42*P2Y2*D40-4*W2*XNUS*KP*D53*P2Y1*D40-4*W2*XNUS*KP*P2Y1*D43Y
X*D40-4*W2*XNUS*KP*P2Y2*D53Y*D40+3*P1Y0*D42**2+2*P1Y0*D42*D43+2*
XP1Y0*D53**2
  FTOUT=FTOUT+2*P1Y0*D41**2+2*P1Y0*D43**2+4*P1Y0*D20**2
C                            TERMS
  FTOUT=
XFTOUT+2*L*W*P1Y0*D43*D20+8*L*W*D41*P1Y1*D20-2*L*W*D43*P1Y2*D20+2*
XL*W*D43Y*P1Y1*D20-2*L*W*D43Y*P1Y3*D20+2*L*D39*D37*W3*P2Y2*XNUS*KP
X-2*L*D39*D37*W3*P2Y0*XNUS*KP+2*L*W3*P2Y2*XNUS*KP*D40*D20-2*L*W3*
XP2Y0*XNUS*KP*D40*D20-2*W*D53*D42*P1Y0+2*W*D53*D42*P1Y2-2*W*D53Y*
XD42*P1Y1+2*W*D53Y*D42*P1Y3-2*W*D42**2*P1Y1+2*W*D42**2*P1Y3-2*W*
XD42*P1Y0*D41+2*W*D42*D41*P1Y2-2*W*D42*D43*P1Y1+2*W*D42*D43*P1Y3-2
X*W*D42*D43Y*P1Y2+2*W*D42*D43Y*P1Y4-2*W*P1Y0*D41*D43-4*W*D41**2*
XP1Y1+2*W*D41*D43*P1Y2-2*W*D41*D43Y*P1Y1+2*W*D41*D43Y*P1Y3-4*W*
XP1Y1*D20**2-2*X**2*W5*D37**2*P5Y1*KP**2-2*X*W5*D37**2*P5Y2*XNUS*
XKP**2
  FTOUT=FTOUT-2*X*W5*D37**2*P5Y0*XNUS*KP**2-2*X*W3*D42*P2Y2*KP*D40+2
X*X*W3*D42*P2Y0*KP*D40-2*W5*D37**2*P5Y1*XNUS**2*KP**2+2*W5*D37**2*
XP4Y1-2*W5*D37**2*P3Y1+2*W3*D42*P2Y1*XNUS*KP*D40-2*W3*D42*P2Y3*
XXNUS*KP*D40-2*W3*P2Y2*D41*XNUS*KP*D40+2*W3*P2Y0*D41*XNUS*KP*D40
```

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C                        TFRM6
      FTOUT=
     XFTOUT+2*L*W*D39**2*P1Y0+2*L*W*D20**2*P1Y0+2*L*W*P1Y1*D42*D43Y+4*L
     X*W*P1Y1*D43*D53+4*L*W*P1Y1*D43*D43Y+4*L*W*P1Y1*D53*D53Y-2*L*W*
     XP1Y2*D42*D43+4*L*W*P1Y2*D43*D53Y+4*L*W*P1Y2*D53*D43Y+2*L*W*P1Y2*
     XD43Y**2+2*L*W*P1Y2*D53Y**2-2*L*W*P1Y3*D42*D43Y+4*L*W*P1Y3*D43Y*
     XD53Y+2*L*W*P1Y0*D42*D43+2*L*W*P1Y0*D43**2+2*L*W*P1Y0*D53**2-2*L*X
     X**2*D37**2*P5Y1*W4*KP**2-2*L*X**2*D37**2*W5*KP**2*P5Y0-4*L*X*D37
     X**2*P5Y1*W5*XNUS*KP**2-2*L*X*D37**2*P5Y2*W4*XNUS*KP**2-2*L*X*D37
     X**2*W4*XNUS*KP**2*P5Y0-4*L*X*W3*KP*P2Y0*D53*D40-4*L*X*W3*KP*D43*
     XP2Y1*D40-4*L*X*W3*KP*D43Y*P2Y2*D40-4*L*X*W3*KP*P2Y1*D53Y*D40+2*L*
     XX*W2*KP*P2Y0*D42*D40
      FTOUT=FTOUT-2*L*X*W2*KP*D42*P2Y2*D40-4*L*X*W2*KP*D43*P2Y2*D40-4*L*
     XX*W2*KP*D53*P2Y1*D40-4*L*X*W2*KP*D43Y*P2Y3*D40-4*L*X*W2*KP*D53Y*
     XP2Y2*D40-2*L*D37**2*P5Y1*W4*XNUS**2*KP**2-2*L*D37**2*P5Y2*W5*XNUS
     X**2*KP**2+2*L*D37**2*W5*P4Y0+2*L*D37**2*W4*P4Y1+2*L*D39**2*P1Y1+2
     X*L*D20**2*P1Y1-2*L*P1Y1*D41*D43Y-2*L*P1Y1*D42*D53Y+2*L*P1Y1*D43**
     X2+2*L*P1Y1*D53**2-2*L*P1Y2*D41*D42+2*L*P1Y2*D41*D43+2*L*P1Y2*D42*
     XD53+4*L*P1Y2*D43*D53+4*L*P1Y2*D43*D43Y+4*L*P1Y2*D53*D53Y+2*L*P1Y3
     X*D41*D43Y+2*L*P1Y3*D42*D53Y+4*L*P1Y3*D43*D53Y+4*L*P1Y3*D53*D43Y+2
     X*L*P1Y3*D43Y**2+2*L*P1Y3*D53Y**2+4*L*P1Y4*D43Y*D53Y+2*L*W5*P3Y0*
     XD40**2
      FTOUT=FTOUT+2*L*W4*P3Y1*D40**2-2*L*W3*XNUS*KP*P2Y0*D42*D40-4*L*W3*
     XXNUS*KP*P2Y0*D43*D40+2*L*W3*XNUS*KP*D42*P2Y2*D40-4*L*W3*XNUS*KP*
     XD53*P2Y1*D40-4*L*W3*XNUS*KP*D43Y*P2Y1*D40-4*L*W3*XNUS*KP*D53Y*
     XP2Y2*D40+2*L*W2*XNUS*KP*P2Y0*D41*D40-2*L*W2*XNUS*KP*D41*P2Y2*D40-
     X4*L*W2*XNUS*KP*D43*P2Y1*D40-4*L*W2*XNUS*KP*D53*P2Y2*D40-4*L*W2*
     XXNUS*KP*D43Y*P2Y2*D40-4*L*W2*XNUS*KP*D53Y*P2Y3*D40+2*L*P1Y0*D41*
     XD42-2*L*P1Y0*D41*D43-2*L*P1Y0*D42*D53+4*W*D20*P1Y1*D43+4*W*D20*
     XP1Y1*D53Y+4*W*D20*P1Y2*D43Y+4*W*D20*P1Y0*D53-4*X*D37*D39*W3*KP*
     XP2Y0-4*X*D37*D39*W2*KP*P2Y1-4*X*D20*W3*KP*P2Y0*D40-4*X*D20*W2*KP*
     XP2Y1*D40
      FTOUT=FTOUT-4*D37*D39*W3*XNUS*KP*P2Y1-2*D37*D39*W2*XNUS*KP*P2Y0-2*
     XD37*D39*W2*XNUS*KP*P2Y2+4*D20*P1Y1*D53+2*D20*P1Y1*D43Y+4*D20*P1Y2
     X*D42+2*D20*P1Y2*D43+4*D20*P1Y2*D53Y+2*D20*P1Y3*D43Y-4*D20*W3*XNUS
     X*KP*P2Y1*D40-2*D20*W2*XNUS*KP*P2Y0*D40-2*D20*W2*XNUS*KP*P2Y2*D40-
```

MS2

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C    1/(M**2) TERMS FROM SQUARE OF 1 * 1/(M**2)
C                        TERM1
      FTOUT=
     XFTOUT-4*L*W*F4*P1Y2*C1+4*L*X*C2*E4*W3*KP*P2Y1-4*L*X*C2*E4*W2*
     XP2Y0*KP+4*L*C2*E4*W3*XNUS*KP*P2Y2-4*L*C2*E4*W2*XNUS*KP*P2Y1+4*L*
     XE4*P1Y1*C1-4*W*E4*C4*P1Y1-4*W*E4*C4Y*P1Y2-2*W*E4*C10*P1Y1+2*W*E4*
     XC10*P1Y3-2*W*E4*C10Y*P1Y2+2*W*E4*C10Y*P1Y4-2*X**2*E4*P8Y1*W5*KP**
     X2*C14+2*X**2*E4*W4*P8Y0*KP**2*C14+4*X*E4*P8Y1*W4*XNUS*KP**2*C14-4
     X*X*E4*W5*P8Y2*XNUS*KP**2*C14+4*X*E4*W3*KP*P2Y1*C7+4*X*F4*W3*KP*
     XP2Y2*C9-4*X*E4*W2*P2Y0*KP*C7-4*X*E4*W2*KP*P2Y1*C9-2*E4*W5*P8Y3*
     XXNUS**2*KP**2*C14+2*E4*W5*P7Y1*C14-2*E4*W5*P6Y1*C14-4*E4*W5*P6Y1*
     XC8+2*E4*P8Y2*W4*XNUS**2*KP**2*C14-2*E4*W4*P7Y0*C14+2*F4*W4*P6Y0*
     XC14
      FTOUT=FTOUT+4*E4*W4*P6Y0*C8+4*E4*W3*XNUS*KP*P2Y1*C9+2*E4*W3*XNUS*
     XKP*P2Y1*C13+2*E4*W3*XNUS*KP*P2Y1*C15+4*E4*W3*XNUS*KP*P2Y2*C7+2*E4
     X*W3*XNUS*KP*P2Y2*C13Y-2*E4*W3*XNUS*KP*P2Y3*C13-2*E4*W3*XNUS*KP*
     XP2Y3*C15-2*E4*W3*XNUS*KP*P2Y4*C13Y-4*E4*W2*P2Y0*XNUS*KP*C9-2*E4*
     XW2*P2Y0*XNUS*KP*C13-2*E4*W2*P2Y0*XNUS*KP*C15-4*E4*W2*XNUS*KP*P2Y1
     X*C7-2*E4*W2*XNUS*KP*P2Y1*C13Y+2*E4*W2*XNUS*KP*P2Y2*C13+2*E4*W2*
     XXNUS*KP*P2Y2*C15+2*E4*W2*XNUS*KP*P2Y3*C13Y+4*E4*P1Y0*C4+2*E4*P1Y0
     X*C10+4*E4*C4Y*P1Y1-2*E4*C10*P1Y2+2*E4*C10Y*P1Y1-2*E4*C10Y*P1Y3
C                        TERM2
      FTOUT=
     XFTOUT-4*L*W*E4*P1Y0*C20-2*L*W*E4*P1Y0*C30+4*L*W*E4*C27*P1Y1-4*L*
     XW*F4*C20Y*P1Y1+2*L*W*E4*C30*P1Y2-4*L*W*E20*P1Y0*C4-2*L*W*E20*P1Y0
     X*C10+2*L*W*E20*C10*P1Y2-4*L*W*E20*C4Y*P1Y1-2*L*W*E20*C10Y*P1Y1+2*
     XL*W*E20*C10Y*P1Y3+2*L*X**2*E4*C33*P8Y0*KP**2*W5-2*L*X**2*E4*C33*
     XP8Y1*KP**2*W4-2*L*X**2*C14*E20*P8Y0*KP**2*W5+2*L*X**2*C14*F20*
     XP8Y1*KP**2*W4-2*L*X*E4*C33*P8Y0*XNUS*KP**2*W4+4*L*X*E4*C33*P8Y1*
     XXNUS*KP**2*W5-2*L*X*E4*C33*P8Y2*XNUS*KP**2*W4+4*L*X*E4*P2Y0*KP*
     XC22*W3+4*L*X*E4*P2Y0*KP*C28*W3+2*L*X*E4*P2Y0*KP*C29*W2-2*L*X*E4*
     XP2Y0*KP*C34*W2-2*L*X*E4*P2Y0*KP*C25*W2-4*L*X*E4*KP*C22*P2Y1*W2-4*
     XL*X*E4*KP*C28*P2Y1*W2
      FTOUT=FTOUT-2*L*X*E4*KP*C29*P2Y2*W2+4*L*X*E4*KP*C24*P2Y1*W3-4*L*X*
     XF4*KP*C24*P2Y2*W2+2*L*X*F4*KP*C34*P2Y2*W2+2*L*X*E4*KP*C25*P2Y2*W2
     X+4*L*X*C9*E20*KP*P2Y1*W3-4*L*X*C9*F20*KP*P2Y2*W2+2*L*X*E39*C3*
```

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XP2Y0*KP*W2-2*L*X*E39*C3*KP*P2Y2*W2+2*L*X*C13Y*E20*KP*P2Y1*W2-2*L*
XX*C13Y*E20*KP*P2Y3*W2+2*L*X*C14*E20*P8Y0*XNUS*KP**2*W4-4*L*X*C14*
XE20*P8Y1*XNUS*KP**2*W5+2*L*X*C14*E20*P8Y2*XNUS*KP**2*W4+2*L*X*F20
X*P2Y0*KP*C13*W2+4*L*X*E20*P2Y0*KP*C7*W3-2*L*X*E20*KP*C13*P2Y2*W2-
X4*L*X*E20*KP*C7*P2Y1*W2-2*L*E4*C33*P7Y0*W5+2*L*E4*C33*P6Y0*W5-2*L
X*E4*C33*P8Y1*XNUS**2*KP**2*W4+2*L*E4*C33*P8Y2*XNUS**2*KP**2*W5+2*
XL*E4*C33*P7Y1*W4
 FTOUT=FTOUT-2*L*E4*C33*P6Y1*W4-4*L*E4*P6Y0*C23*W5+2*L*E4*P2Y0*C26*
XXNUS*KP*W2+2*L*E4*P2Y0*XNUS*KP*C29*W3+4*L*E4*P2Y0*XNUS*KP*C24*W3-
X2*L*E4*P2Y0*XNUS*KP*C34*W3+2*L*E4*P2Y0*XNUS*KP*C32*W3+2*L*E4*P2Y0
X*XNUS*KP*C38*W3-2*L*E4*P1Y0*C31+4*L*E4*C20*P1Y1-4*L*E4*C27*P1Y2+4
X*L*E4*C20Y*P1Y2+2*L*E4*C31*P1Y2-2*L*E4*C26*XNUS*KP*P2Y2*W2+4*L*E4
X*XNUS*KP*C22*P2Y1*W3-4*L*E4*XNUS*KP*C22*P2Y2*W2+4*L*E4*XNUS*KP*
XC28*P2Y1*W3-4*L*E4*XNUS*KP*C28*P2Y2*W2-2*L*E4*XNUS*KP*C29*P2Y2*W3
X-4*L*E4*XNUS*KP*C24*P2Y1*W2+2*L*E4*XNUS*KP*C34*P2Y2*W3-2*L*E4*
XXNUS*KP*C32*P2Y2*W3-2*L*E4*XNUS*KP*C38*P2Y2*W3+4*L*E4*C23*P6Y1*W4
X+4*L*C9*E20*P2Y0*XNUS*KP*W3
 FTOUT=FTOUT-4*L*C9*E20*XNUS*KP*P2Y1*W2+2*L*C13Y*E20*XNUS*KP*P2Y1*
XW3-2*L*C13Y*E20*XNUS*KP*P2Y3*W3+2*L*C15*E20*P2Y0*XNUS*KP*W3-2*L*
XC15*E20*XNUS*KP*P2Y2*W3+2*L*C14*E20*P7Y0*W5-2*L*C14*E20*P6Y0*W5+2
X*L*C14*E20*P8Y1*XNUS**2*KP**2*W4-2*L*C14*E20*P8Y2*XNUS**2*KP**2*
XW5-2*L*C14*E20*P7Y1*W4+2*L*C14*E20*P6Y1*W4-4*L*E20*C8*P6Y0*W5+4*L
X*E20*C8*P6Y1*W4+2*L*E20*P2Y0*XNUS*KP*C13*W3+2*L*E20*P2Y0*XNUS*KP*
XC16*W2-2*L*E20*P1Y0*C11+4*L*E20*C4*P1Y1+2*L*E20*C11*P1Y2-2*L*E20*
XXNUS*KP*C13*P2Y2*W3-2*L*E20*XNUS*KP*C16*P2Y2*W2+4*L*E20*XNUS*KP*
XC7*P2Y1*W3-4*L*E20*XNUS*KP*C7*P2Y2*W2+4*L*E20*C4Y*P1Y2-2*L*E20*
XC11Y*P1Y1
 FTOUT=FTOUT+2*L*E20*C11Y*P1Y3+4*W*E4*C41*P1Y0-4*W*E20*C1*P1Y1-4*X*
XE4*P2Y0*C44*KP*W3+4*X*E4*C44*KP*P2Y1*W2+2*X*E39*P8Y0*C17*XNUS*KP
X**2*W4-2*X*E39*C17*P8Y2*XNUS*KP**2*W4+4*X*E20*P2Y0*C2*KP*W3-4*X*
XE20*C2*KP*P2Y1*W2-4*E4*C41*P1Y1-2*E4*C43*P1Y0+2*E4*C43*P1Y2+2*E4*
XP2Y0*C40*XNUS*KP*W2+4*E4*P2Y0*C44*XNUS*KP*W2+2*E4*P2Y0*C48*XNUS*
XKP*W2-2*E4*P1Y0*C42+2*E4*C42*P1Y2-2*E4*C43Y*P1Y1+2*E4*C43Y*P1Y3-2
X*E4*C40*XNUS*KP*P2Y2*W2-4*E4*C44*XNUS*KP*P2Y1*W3-2*E4*C48*XNUS*KP
X*P2Y2*W2+2*E4*C40Y*XNUS*KP*P2Y1*W2-2*E4*C40Y*XNUS*KP*P2Y3*W2+2*
XE39*P2Y0*C5*XNUS*KP*W2-2*E39*P1Y0*C6+2*E39*C6*P1Y2-2*E39*C5*XNUS*
```

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XKP*P2Y2*W2
 FTOUT=FTOUT+2*E39*C17*P8Y1*XNUS**2*KP**2*W4-2*E39*C17*P8Y3*XNUS**2
X*KP**2*W4-2*E20*P2Y0*C2*XNUS*KP*W2+2*E20*P1Y0*C1+2*E20*C1*P1Y2+4*
XE20*C2*XNUS*KP*P2Y1*W3-2*E20*C2*XNUS*KP*P2Y2*W2
C                    TERM3
 FTOUT=
XFTOUT-2*L*W*E39*P1Y0*C6+2*L*W*E39*C6*P1Y2-2*L*W*C43*E4*P1Y0+2*L*
XW*C43*E4*P1Y2+2*L*W*C42*E4*P1Y0-2*L*W*C42*E4*P1Y2-2*L*W*C43Y*E4*
XP1Y1+2*L*W*C43Y*E4*P1Y3+2*L*W*P1Y0*C1*E20-2*L*W*C1*E20*P1Y2-2*L*X
X*C17*E39*XNUS*KP**2*P8Y2*W5+2*L*X*C17*E39*XNUS*KP**2*P8Y0*W5+2*L*
XC17*E39*XNUS**2*P8Y1*KP**2*W5-2*L*C17*E39*XNUS**2*KP**2*P8Y3*W5-2
X*L*E39*XNUS*KP*P2Y2*C5*W3+2*L*E39*XNUS*KP*P2Y0*C5*W3-2*L*XNUS*KP*
XC40*E4*P2Y2*W3+2*L*XNUS*KP*C40*E4*P2Y0*W3+2*L*XNUS*KP*C48*E4*P2Y2
X*W3-2*L*XNUS*KP*C48*E4*P2Y0*W3+2*L*XNUS*KP*C40Y*E4*P2Y1*W3-2*L*
XXNUS*KP*C40Y*E4*P2Y3*W3+2*L*XNUS*KP*P2Y2*E20*C2*W3-2*L*XNUS*KP*
XP2Y0*E20*C2*W3
 FTOUT=FTOUT+2*W*E4*C31*P1Y0-2*W*E4*C31*P1Y2+2*W*E4*C30*P1Y1-2*W*E4
X*C30*P1Y3-2*W*C11*P1Y0*E20+2*W*C11*E20*P1Y2-2*W*C10*E20*P1Y1+2*W*
XC10*E20*P1Y3-2*W*C11Y*E20*P1Y1+2*W*C11Y*E20*P1Y3-2*W*C10Y*E20*
XP1Y2+2*W*C10Y*E20*P1Y4+2*X*C33*XNUS*KP**2*P8Y2*E4*W5-2*X*C33*XNUS
X*KP**2*E4*P8Y0*W5-2*X*E39*KP*P2Y2*C3*W3+2*X*E39*KP*C3*P2Y0*W3-2*X
X*XNUS*KP**2*P8Y2*C14*E20*W5+2*X*XNUS*KP**2*P8Y0*C14*E20*W5+2*X*KP
X*E4*C29*P2Y2*W3-2*X*KP*E4*C29*P2Y0*W3+2*X*KP*E4*C34*P2Y2*W3-2*X*
XKP*E4*C34*P2Y0*W3+2*X*KP*E4*P2Y2*C25*W3-2*X*KP*E4*P2Y0*C25*W3-2*X
X*KP*C13*P2Y2*E20*W3+2*X*KP*C13*P2Y0*E20*W3+2*X*KP*C13Y*P2Y1*E20*
XW3
 FTOUT=FTOUT-2*X*KP*C13Y*P2Y3*E20*W3-2*C33*XNUS**2*P8Y1*KP**2*E4*W5
X+2*C33*XNUS**2*KP**2*P8Y3*E4*W5+2*XNUS**2*P8Y1*KP**2*C14*E20*W5-2
X*XNUS**2*KP**2*P8Y3*C14*E20*W5-2*XNUS*KP*E4*C29*P2Y1*W3+2*XNUS*KP
X*E4*C29*P2Y3*W3-2*XNUS*KP*E4*C34*P2Y1*W3+2*XNUS*KP*E4*C34*P2Y3*W3
X-2*XNUS*KP*E4*C26*P2Y2*W3+2*XNUS*KP*E4*C26*P2Y0*W3+2*XNUS*KP*E4*
XP2Y1*C32*W3-2*XNUS*KP*E4*P2Y1*C38*W3-2*XNUS*KP*E4*P2Y3*C32*W3+2*
XXNUS*KP*E4*P2Y3*C38*W3+2*XNUS*KP*C13*P2Y1*E20*W3-2*XNUS*KP*C13*
XP2Y3*E20*W3+2*XNUS*KP*C13Y*P2Y2*E20*W3-2*XNUS*KP*C13Y*P2Y4*E20*W3
X-2*XNUS*KP*C16*P2Y2*E20*W3+2*XNUS*KP*C16*P2Y0*E20*W3+2*XNUS*KP*
XC15*P2Y1*E20*W3
```

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      FTOUT=FTOUT-2*XNUS*KP*C15*P2Y3*E20*W3
C                          TERM4
      FTOUT=
     XFTOUT+4*L*W*E20*C53*P1Y1-4*L*W*E20*C41*P1Y1+4*L*W*E20*C43*P1Y2+4*
     XL*W*E20*C53Y*P1Y2+4*L*W*E20*C43Y*P1Y3+4*L*W*C21*E39*P1Y2-4*L*X*
     XC40Y*E20*P2Y1*W2*KP-4*L*X*C40Y*E20*P2Y2*W3*KP+4*L*X*E20*P2Y1*C44*
     XW3*KP-4*L*X*E20*P2Y1*C40*W3*KP+4*L*X*E20*C44*W2*P2Y0*KP-4*L*X*E20
     X*C40*W2*P2Y0*KP-4*L*X*C37*P2Y1*E39*W3*KP-4*L*X*C37*E39*W2*P2Y0*KP
     X-4*L*C40Y*E20*P2Y2*W2*XNUS*KP-4*L*C40Y*E20*P2Y3*W3*XNUS*KP+4*L*
     XE20*P2Y1*C44*W2*XNUS*KP-4*L*E20*P2Y1*C40*W2*XNUS*KP+4*L*E20*P2Y2*
     XC44*W3*XNUS*KP-4*L*E20*P2Y2*C40*W3*XNUS*KP+4*L*E20*C53*P1Y0-4*L*
     XE20*C41*P1Y0+4*L*E20*C43*P1Y1+4*L*E20*C53Y*P1Y1+4*L*E20*C43Y*P1Y2
     X+4*L*C21*E39*P1Y1
      FTOUT=FTOUT-4*L*C37*P2Y1*E39*W2*XNUS*KP-4*L*C37*P2Y2*E39*W3*XNUS*
     XKP+8*W*E20*C20*P1Y1+8*W*E20*C20Y*P1Y2-4*W*E20*C27*P1Y2+4*W*E20*
     XC30*P1Y1-4*W*E20*C30*P1Y3+4*W*C39*E39*P1Y1+4*W*C39Y*E39*P1Y2+2*X
     X**2*E20*P8Y1*C33*W5*KP**2+2*X**2*E20*C33*W4*P8Y0*KP**2+2*X**2*
     XP8Y1*C52*E39*W5*KP**2+2*X**2*C52*E39*W4*P8Y0*KP**2+4*X*E20*P8Y1*
     XC33*W4*XNUS*KP**2+4*X*E20*P8Y2*C33*W5*XNUS*KP**2-8*X*E20*C22*P2Y1
     X*W3*KP-8*X*E20*C22*W2*P2Y0*KP-8*X*E20*C24*P2Y1*W2*KP-8*X*E20*C24*
     XP2Y2*W3*KP-4*X*E20*C28*P2Y1*W3*KP-4*X*E20*C28*W2*P2Y0*KP+4*X*E20*
     XC35*P2Y1*W3*KP+4*X*E20*C35*W2*P2Y0*KP-4*X*E20*C32*P2Y1*W2*KP-4*X*
     XE20*C32*P2Y2*W3*KP
      FTOUT=FTOUT+4*X*E20*P2Y1*C34*W2*KP+4*X*E20*C34*P2Y2*W3*KP+4*X*P8Y1
     X*C52*E39*W4*XNUS*KP**2+4*X*P8Y2*C52*E39*W5*XNUS*KP**2-4*X*C45*
     XP2Y1*E39*W3*KP-4*X*C45*E39*W2*P2Y0*KP-4*X*C47*P2Y1*E39*W2*KP-4*X*
     XC47*P2Y2*E39*W3*KP+2*E20*P8Y2*C33*W4*XNUS**2*KP**2+2*E20*P8Y3*C33
     X*W5*XNUS**2*KP**2-2*E20*P7Y1*C33*W5+8*E20*C20*P1Y0+8*E20*C20Y*
     XP1Y1-4*E20*C27*P1Y1-4*E20*C30*P1Y2+4*E20*C30*P1Y0-8*E20*C22*P2Y1*
     XW2*XNUS*KP-8*E20*C22*P2Y2*W3*XNUS*KP+8*E20*P6Y1*C23*W5-2*E20*P6Y1
     X*C33*W5-8*E20*C24*P2Y1*W3*XNUS*KP-8*E20*C24*W2*P2Y0*XNUS*KP-4*E20
     X*C28*P2Y1*W2*XNUS*KP-4*E20*C28*P2Y2*W3*XNUS*KP+4*E20*C35*P2Y1*W2*
     XXNUS*KP
      FTOUT=FTOUT+4*E20*C35*P2Y2*W3*XNUS*KP-2*E20*C32*P2Y1*W3*XNUS*KP-2*
     XE20*C32*P2Y2*W2*XNUS*KP-2*E20*C32*P2Y3*W3*XNUS*KP-2*E20*C32*W2*
     XP2Y0*XNUS*KP-4*E20*C38*P2Y1*W3*XNUS*KP+4*E20*C38*P2Y2*W2*XNUS*KP+
```

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     X4*E20*C38*P2Y3*W3*XNUS*KP-4*E20*C38*W2*P2Y0*XNUS*KP-4*E20*C29*
     XP2Y1*W3*XNUS*KP+4*E20*C29*P2Y2*W2*XNUS*KP+4*E20*C29*P2Y3*W3*XNUS*
     XKP-4*E20*C29*W2*P2Y0*XNUS*KP+2*E20*P2Y1*C34*W3*XNUS*KP+2*E20*C34*
     XP2Y2*W2*XNUS*KP+2*E20*C34*P2Y3*W3*XNUS*KP+2*E20*C34*W2*P2Y0*XNUS*
     XKP+8*E20*C23*W4*P6Y0-2*E20*C33*W4*P7Y0-2*E20*C33*W4*P6Y0-2*C51*
     XP2Y1*E39*W3*XNUS*KP+2*C51*P2Y2*E39*W2*XNUS*KP+2*C51*P2Y3*E39*W3*
     XXNUS*KP
      FTOUT=FTOUT-2*C51*E39*W2*P2Y0*XNUS*KP+2*P8Y2*C52*E39*W4*XNUS**2*KP
     X**2+2*P8Y3*C52*E39*W5*XNUS**2*KP**2-2*C52*P7Y1*E39*W5+2*C52*P6Y1*
     XE39*W5-2*C52*E39*W4*P7Y0+2*C52*E39*W4*P6Y0+4*C39*E39*P1Y0+4*C39Y*
     XE39*P1Y1+4*P6Y1*C46*E39*W5-4*C45*P2Y1*E39*W2*XNUS*KP-4*C45*P2Y2*
     XE39*W3*XNUS*KP-4*C47*P2Y1*E39*W3*XNUS*KP-4*C47*E39*W2*P2Y0*XNUS*
     XKP+4*C46*E39*W4*P6Y0
C                            TERM5
      FTOUT=
     XFTOUT+8*L*W*E20*C41*P1Y1+2*L*W*E20*C43*P1Y0-2*L*W*E20*C43*P1Y2+2*
     XL*W*E20*C43Y*P1Y1-2*L*W*E20*C43Y*P1Y3+2*L*W*C21*E39*P1Y0-2*L*W*
     XC21*E39*P1Y2-8*L*X*E20*W3*C44*KP*P2Y1-2*L*C40Y*E20*W3*XNUS*KP*
     XP2Y1+2*L*C40Y*E20*W3*XNUS*KP*P2Y3-4*L*E20*W3*C44*XNUS*KP*P2Y2-4*L
     X*E20*W3*C44*XNUS*KP*P2Y0+2*L*E20*W3*C40*XNUS*KP*P2Y2-2*L*E20*W3*
     XC40*XNUS*KP*P2Y0+2*L*W3*C37*E39*XNUS*KP*P2Y2-2*L*W3*C37*E39*XNUS*
     XKP*P2Y0-2*W*E20*C30*P1Y1+2*W*E20*C30*P1Y3-8*W*E20*C20*P1Y1-8*W*
     XE20*C20Y*P1Y2+2*W*E20*C27*P1Y0+6*W*E20*C27*P1Y2+2*W*E20*C31*P1Y0-
     X2*W*E20*C31*P1Y2+4*X**2*E20*W5*C33*P8Y1*KP**2+4*X*E20*W5*C33*P8Y2
     X*XNUS*KP**2
      FTOUT=FTOUT+4*X*E20*W5*C33*XNUS*KP**2*P8Y0+2*X*E20*W3*C32*KP*P2Y2-
     X2*X*E20*W3*C32*KP*P2Y0+8*X*E20*W3*C22*KP*P2Y1+8*X*E20*W3*C24*KP*
     XP2Y2+8*X*E20*W3*C28*KP*P2Y1-2*X*E20*W3*C25*KP*P2Y2+2*X*E20*W3*C25
     X*KP*P2Y0+2*X*E20*W3*C29*KP*P2Y2-2*X*E20*W3*C29*KP*P2Y0-2*X*E20*W3
     X*C34*KP*P2Y2+2*X*E20*W3*C34*KP*P2Y0-2*X*W5*C52*E39*P8Y2*XNUS*KP**
     X2+2*X*W5*C52*E39*XNUS*KP**2*P8Y0-2*X*W3*C51*E39*KP*P2Y2+2*X*W3*
     XC51*E39*KP*P2Y0-8*E20*W5*C23*P6Y1+4*E20*W5*C33*P8Y1*XNUS**2*KP**2
     X-4*E20*W5*C33*P7Y1+4*E20*W5*C33*P6Y1+2*E20*W3*C38*XNUS*KP*P2Y1-2*
     XE20*W3*C38*XNUS*KP*P2Y3+8*E20*W3*C22*XNUS*KP*P2Y2+2*E20*W3*C26*
     XXNUS*KP*P2Y2
      FTOUT=FTOUT-2*E20*W3*C26*XNUS*KP*P2Y0+8*E20*W3*C24*XNUS*KP*P2Y1+6*
```

```
XE20*W3*C28*XNUS*KP*P2Y2+2*E20*W3*C28*XNUS*KP*P2Y0-2*E20*W3*C35*
XXNUS*KP*P2Y2+2*E20*W3*C35*XNUS*KP*P2Y0+2*E20*W3*C29*XNUS*KP*P2Y1-
X2*E20*W3*C29*XNUS*KP*P2Y3+2*W5*C52*E39*P8Y1*XNUS**2*KP**2-2*W5*
XC52*E39*P8Y3*XNUS**2*KP**2-2*W3*C50*E39*XNUS*KP*P2Y2+2*W3*C50*E39
X*XNUS*KP*P2Y0+2*W3*C51*E39*XNUS*KP*P2Y1-2*W3*C51*E39*XNUS*KP*P2Y3
C                   TERM6
 FTOUT=
XFTOUT+4*L*W*E20*C20*P1Y0+4*L*W*E20*C20Y*P1Y1+2*L*W*E20*C30*P1Y0-2
X*L*W*E20*C30*P1Y2+4*L*W*C39*E39*P1Y0+4*L*W*C39Y*E39*P1Y1+4*L*X**2
X*C33*E20*KP**2*P8Y1*W4+4*L*X**2*C33*E20*KP**2*P8Y0*W5+2*L*X**2*KP
X**2*C52*E39*P8Y1*W4+2*L*X**2*KP**2*C52*E39*P8Y0*W5-4*L*X*W3*E20*
XC22*KP*P2Y0-4*L*X*W3*E20*C24*KP*P2Y1+4*L*X*W3*E20*KP*C35*P2Y0+4*L
X*X*W3*E20*KP*C34*P2Y1-4*L*X*W3*E20*KP*C32*P2Y1-4*L*X*W3*C45*KP*
XE39*P2Y0-4*L*X*W3*C47*KP*E39*P2Y1+8*L*X*C33*E20*XNUS*KP**2*P8Y1*
XW5+4*L*X*C33*E20*XNUS*KP**2*P8Y2*W4+4*L*X*C33*E20*XNUS*KP**2*P8Y0
X*W4-4*L*X*E20*W2*C22*KP*P2Y1-4*L*X*E20*W2*C24*KP*P2Y2+2*L*X*E20*
XW2*KP*C29*P2Y0
 FTOUT=FTOUT-2*L*X*E20*W2*KP*C29*P2Y2+4*L*X*E20*W2*KP*C35*P2Y1+2*L*
XX*E20*W2*KP*C34*P2Y0+2*L*X*E20*W2*KP*C34*P2Y2+2*L*X*E20*W2*KP*C25
X*P2Y0-2*L*X*E20*W2*KP*C25*P2Y2-2*L*X*E20*W2*KP*C32*P2Y0-2*L*X*E20
X*W2*KP*C32*P2Y2-4*L*X*W2*C45*KP*E39*P2Y1-4*L*X*W2*C47*KP*E39*P2Y2
X+2*L*X*W2*C51*KP*E39*P2Y0-2*L*X*W2*C51*KP*E39*P2Y2+4*L*X*XNUS*KP
X**2*C52*E39*P8Y1*W5+2*L*X*XNUS*KP**2*C52*E39*P8Y2*W4+2*L*X*XNUS*
XKP**2*C52*E39*P8Y0*W4-4*L*W3*E20*C22*XNUS*KP*P2Y1-4*L*W3*E20*C24*
XXNUS*KP*P2Y0-2*L*W3*E20*XNUS*KP*C29*P2Y0+2*L*W3*E20*XNUS*KP*C29*
XP2Y2+4*L*W3*E20*XNUS*KP*C35*P2Y1+4*L*W3*E20*XNUS*KP*C34*P2Y2-4*L*
XW3*E20*XNUS*KP*C32*P2Y2
 FTOUT=FTOUT-2*L*W3*E20*XNUS*KP*C38*P2Y0+2*L*W3*E20*XNUS*KP*C38*
XP2Y2-4*L*W3*C45*XNUS*KP*E39*P2Y1-4*L*W3*C47*XNUS*KP*E39*P2Y0-2*L*
XW3*XNUS*C51*KP*E39*P2Y0+2*L*W3*XNUS*C51*KP*E39*P2Y2+4*L*C33*E20*
XXNUS**2*KP**2*P8Y1*W4+4*L*C33*E20*XNUS**2*KP**2*P8Y2*W5-4*L*C33*
XE20*P7Y1*W4-4*L*C33*E20*P7Y0*W5-4*L*E20*W2*C22*XNUS*KP*P2Y2-4*L*
XE20*W2*C24*XNUS*KP*P2Y1+2*L*E20*W2*XNUS*C28*KP*P2Y0-2*L*E20*W2*
XXNUS*C28*KP*P2Y2+2*L*E20*W2*XNUS*KP*C35*P2Y0+2*L*E20*W2*XNUS*KP*
XC35*P2Y2-2*L*E20*W2*XNUS*KP*C26*P2Y0+2*L*E20*W2*XNUS*KP*C26*P2Y2+
X4*L*E20*W2*XNUS*KP*C34*P2Y1-4*L*E20*W2*XNUS*KP*C32*P2Y1+4*L*E20*
```

```
XC20*P1Y1
 FTOUT=FTOUT+4*L*E20*C20Y*P1Y2+2*L*F20*C27*P1Y0-2*L*E20*C27*P1Y2-2*
XL*E20*C31*P1Y0+2*L*E20*C31*P1Y2+4*L*E20*C23*P6Y1*W4+4*L*E20*C23*
XP6Y0*W5+2*L*W2*C50*XNUS*KP*E39*P2Y0-2*L*W2*C50*XNUS*KP*E39*P2Y2-4
X*L*W2*C45*XNUS*KP*E39*P2Y2-4*L*W2*C47*XNUS*KP*E39*P2Y1+4*L*C39*
XE39*P1Y1+4*L*C39Y*E39*P1Y2+2*L*XNUS**2*KP**2*C52*E39*P8Y1*W4+2*L*
XXNUS**2*KP**2*C52*E39*P8Y2*W5-2*L*C52*E39*P7Y1*W4+2*L*C52*E39*
XP6Y1*W4-2*L*C52*E39*P7Y0*W5+2*L*C52*E39*P6Y0*W5+4*L*C46*E39*P6Y1*
XW4+4*L*C46*E39*P6Y0*W5+4*W*E20*C53*P1Y0+4*W*E20*C43*P1Y1+4*W*E20*
XC53Y*P1Y1+4*W*E20*C43Y*P1Y2+4*W*C21*E39*P1Y1-4*X*W3*E20*C40*KP*
XP2Y0
 FTOUT=FTOUT-4*X*W3*E20*C40Y*KP*P2Y1-4*X*W3*KP*C37*E39*P2Y0-4*X*E20
X*W2*C40*KP*P2Y1-4*X*F20*W2*C40Y*KP*P2Y2-4*X*W2*KP*C37*E39*P2Y1-4*
XW3*E20*C40*XNUS*KP*P2Y1-4*W3*E20*C40Y*XNUS*KP*P2Y2-4*W3*XNUS*KP*
XC37*E39*P2Y1+4*E20*W2*C48*XNUS*KP*P2Y0-4*E20*W2*C48*XNUS*KP*P2Y2-
X2*E20*W2*C40*XNUS*KP*P2Y0-2*E20*W2*C40*XNUS*KP*P2Y2-2*E20*W2*C40Y
X*XNUS*KP*P2Y1-2*E20*W2*C40Y*XNUS*KP*P2Y3+4*E20*C53*P1Y1-4*E20*C42
X*P1Y0+4*E20*C42*P1Y2+2*E20*C43*P1Y0+2*E20*C43*P1Y2+4*E20*C53Y*
XP1Y2+2*E20*C43Y*P1Y1+2*E20*C43Y*P1Y3-2*W2*XNUS*KP*C37*E39*P2Y0-2*
XW2*XNUS*KP*C37*E39*P2Y2+2*C21*E39*P1Y0+2*C21*E39*P1Y2
```

```
C    1/(M**3) TERMS FROM SQUARE OF 1/M * 1/(M**2)
C    FIRST SET OF TERMS CORRESPOND TO THOSE DERIVED FROM 1 * 1/(M**2)
C    VIA THE REPLACEMENT E -> D
C                        TERM1
      FTOUT=
     XFTOUT-4*L*W*D4*P1Y2*C1+4*L*X*C2*D4*W3*KP*P2Y1-4*L*X*C2*D4*W2*
     XP2Y0*KP+4*L*C2*D4*W3*XNUS*KP*P2Y2-4*L*C2*D4*W2*XNUS*KP*P2Y1+4*L*
     XD4*P1Y1*C1-4*W*D4*C4*P1Y1-4*W*D4*C4Y*P1Y2-2*W*D4*C10*P1Y1+2*W*D4*
     XC10*P1Y3-2*W*D4*C10Y*P1Y2+2*W*D4*C10Y*P1Y4-2*X**2*D4*P8Y1*W5*KP**
     X2*C14+2*X**2*D4*W4*P8Y0*KP**2*C14+4*X*D4*P8Y1*W4*XNUS*KP**2*C14-4
     X*X*D4*W5*P8Y2*XNUS*KP**2*C14+4*X*D4*W3*KP*P2Y1*C7+4*X*D4*W3*KP*
     XP2Y2*C9-4*X*D4*W2*P2Y0*KP*C7-4*X*D4*W2*KP*P2Y1*C9-2*D4*W5*P8Y3*
     XXNUS**2*KP**2*C14+2*D4*W5*P7Y1*C14-2*D4*W5*P6Y1*C14-4*D4*W5*P6Y1*
     XC8+2*D4*P8Y2*W4*XNUS**2*KP**2*C14-2*D4*W4*P7Y0*C14+2*D4*W4*P6Y0*
     XC14
      FTOUT=FTOUT+4*D4*W4*P6Y0*C8+4*D4*W3*XNUS*KP*P2Y1*C9+2*D4*W3*XNUS*
     XKP*P2Y1*C13+2*D4*W3*XNUS*KP*P2Y1*C15+4*D4*W3*XNUS*KP*P2Y2*C7+2*D4
     X*W3*XNUS*KP*P2Y2*C13Y-2*D4*W3*XNUS*KP*P2Y3*C13-2*D4*W3*XNUS*KP*
     XP2Y3*C15-2*D4*W3*XNUS*KP*P2Y4*C13Y-4*D4*W2*P2Y0*XNUS*KP*C9-2*D4*
     XW2*P2Y0*XNUS*KP*C13-2*D4*W2*P2Y0*XNUS*KP*C15-4*D4*W2*XNUS*KP*P2Y1
     X*C7-2*D4*W2*XNUS*KP*P2Y1*C13Y+2*D4*W2*XNUS*KP*P2Y2*C13+2*D4*W2*
     XXNUS*KP*P2Y2*C15+2*D4*W2*XNUS*KP*P2Y3*C13Y+4*D4*P1Y0*C4+2*D4*P1Y0
     X*C10+4*D4*C4Y*P1Y1-2*D4*C10*P1Y2+2*D4*C10Y*P1Y1-2*D4*C10Y*P1Y3
C                        TERM2
      FTOUT=
     XFTOUT-4*L*W*D4*P1Y0*C20-2*L*W*D4*P1Y0*C30+4*L*W*D4*C27*P1Y1-4*L*
     XW*D4*C20Y*P1Y1+2*L*W*D4*C30*P1Y2-4*L*W*D20*P1Y0*C4-2*L*W*D20*P1Y0
     X*C10+2*L*W*D20*C10*P1Y2-4*L*W*D20*C4Y*P1Y1-2*L*W*D20*C10Y*P1Y1+2*
     XL*W*D20*C10Y*P1Y3+2*L*X**2*D4*C33*P8Y0*KP**2*W5-2*L*X**2*D4*C33*
     XP8Y1*KP**2*W4-2*L*X**2*C14*D20*P8Y0*KP**2*W5+2*L*X**2*C14*D20*
     XP8Y1*KP**2*W4-2*L*X*D4*C33*P8Y0*XNUS*KP**2*W4+4*L*X*D4*C33*P8Y1*
     XXNUS*KP**2*W5-2*L*X*D4*C33*P8Y2*XNUS*KP**2*W4+4*L*X*D4*P2Y0*KP*
     XC22*W3+4*L*X*D4*P2Y0*KP*C28*W3+2*L*X*D4*P2Y0*KP*C29*W2-2*L*X*D4*
     XP2Y0*KP*C34*W2-2*L*X*D4*P2Y0*KP*C25*W2-4*L*X*D4*KP*C22*P2Y1*W2-4*
     XL*X*D4*KP*C28*P2Y1*W2
      FTOUT=FTOUT-2*L*X*D4*KP*C29*P2Y2*W2+4*L*X*D4*KP*C24*P2Y1*W3-4*L*X*
```

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XD4*KP*C24*P2Y2*W2+2*L*X*D4*KP*C34*P2Y2*W2+2*L*X*D4*KP*C25*P2Y2*W2
X+4*L*X*C9*D20*KP*P2Y1*W3-4*L*X*C9*D20*KP*P2Y2*W2+2*L*X*D39*C3*
XP2Y0*KP*W2-2*L*X*D39*C3*KP*P2Y2*W2+2*L*X*C13Y*D20*KP*P2Y1*W2-2*L*
XX*C13Y*D20*KP*P2Y3*W2+2*L*X*C14*D20*P8Y0*XNUS*KP**2*W4-4*L*X*C14*
XD20*P8Y1*XNUS*KP**2*W5+2*L*X*C14*D20*P8Y2*XNUS*KP**2*W4+2*L*X*D20
X*P2Y0*KP*C13*W2+4*L*X*D20*P2Y0*KP*C7*W3-2*L*X*D20*KP*C13*P2Y2*W2-
X4*L*X*D20*KP*C7*P2Y1*W2-2*L*D4*C33*P7Y0*W5+2*L*D4*C33*P6Y0*W5-2*L
X*D4*C33*P8Y1*XNUS**2*KP**2*W4+2*L*D4*C33*P8Y2*XNUS**2*KP**2*W5+2*
XL*D4*C33*P7Y1*W4
 FTOUT=FTOUT-2*L*D4*C33*P6Y1*W4-4*L*D4*P6Y0*C23*W5+2*L*D4*P2Y0*C26*
XXNUS*KP*W2+2*L*D4*P2Y0*XNUS*KP*C29*W3+4*L*D4*P2Y0*XNUS*KP*C24*W3-
X2*L*D4*P2Y0*XNUS*KP*C34*W3+2*L*D4*P2Y0*XNUS*KP*C32*W3+2*L*D4*P2Y0
X*XNUS*KP*C38*W3-2*L*D4*P1Y0*C31+4*L*D4*C20*P1Y1-4*L*D4*C27*P1Y2+4
X*L*D4*C20Y*P1Y2+2*L*D4*C31*P1Y2-2*L*D4*C26*XNUS*KP*P2Y2*W2+4*L*D4
X*XNUS*KP*C22*P2Y1*W3-4*L*D4*XNUS*KP*C22*P2Y2*W2+4*L*D4*XNUS*KP*
XC28*P2Y1*W3-4*L*D4*XNUS*KP*C28*P2Y2*W2-2*L*D4*XNUS*KP*C29*P2Y2*W3
X-4*L*D4*XNUS*KP*C24*P2Y1*W2+2*L*D4*XNUS*KP*C34*P2Y2*W3-2*L*D4*
XXNUS*KP*C32*P2Y2*W3-2*L*D4*XNUS*KP*C38*P2Y2*W3+4*L*D4*C23*P6Y1*W4
X+4*L*C9*D20*P2Y0*XNUS*KP*W3
 FTOUT=FTOUT-4*L*C9*D20*XNUS*KP*P2Y1*W2+2*L*C13Y*D20*XNUS*KP*P2Y1*
XW3-2*L*C13Y*D20*XNUS*KP*P2Y3*W3+2*L*C15*D20*P2Y0*XNUS*KP*W3-2*L*
XC15*D20*XNUS*KP*P2Y2*W3+2*L*C14*D20*P7Y0*W5-2*L*C14*D20*P6Y0*W5+2
X*L*C14*D20*P8Y1*XNUS**2*KP**2*W4-2*L*C14*D20*P8Y2*XNUS**2*KP**2*
XW5-2*L*C14*D20*P7Y1*W4+2*L*C14*D20*P6Y1*W4-4*L*D20*C8*P6Y0*W5+4*L
X*D20*C8*P6Y1*W4+2*L*D20*P2Y0*XNUS*KP*C13*W3+2*L*D20*P2Y0*XNUS*KP*
XC16*W2-2*L*D20*P1Y0*C11+4*L*D20*C4*P1Y1+2*L*D20*C11*P1Y2-2*L*D20*
XXNUS*KP*C13*P2Y2*W3-2*L*D20*XNUS*KP*C16*P2Y2*W2+4*L*D20*XNUS*KP*
XC7*P2Y1*W3-4*L*D20*XNUS*KP*C7*P2Y2*W2+4*L*D20*C4Y*P1Y2-2*L*D20*
XC11Y*P1Y1
 FTOUT=FTOUT+2*L*D20*C11Y*P1Y3+4*W*D4*C41*P1Y0-4*W*D20*C1*P1Y1-4*X*
XD4*P2Y0*C44*KP*W3+4*X*D4*C44*KP*P2Y1*W2+2*X*D39*P8Y0*C17*XNUS*KP
X**2*W4-2*X*D39*C17*P8Y2*XNUS*KP**2*W4+4*X*D20*P2Y0*C2*KP*W3-4*X*
XD20*C2*KP*P2Y1*W2-4*D4*C41*P1Y1-2*D4*C43*P1Y0+2*D4*C43*P1Y2+2*D4*
XP2Y0*C40*XNUS*KP*W2+4*D4*P2Y0*C44*XNUS*KP*W2+2*D4*P2Y0*C48*XNUS*
XKP*W2-2*D4*P1Y0*C42+2*D4*C42*P1Y2-2*D4*C43Y*P1Y1+2*D4*C43Y*P1Y3-2
X*D4*C40*XNUS*KP*P2Y2*W2-4*D4*C44*XNUS*KP*P2Y1*W3-2*D4*C48*XNUS*KP
```

```
     X*P2Y2*W2+2*D4*C40Y*XNUS*KP*P2Y1*W2-2*D4*C40Y*XNUS*KP*P2Y3*W2+2*
     XD39*P2Y0*C5*XNUS*KP*W2-2*D39*P1Y0*C6+2*D39*C6*P1Y2-2*D39*C5*XNUS*
     XKP*P2Y2*W2
      FTOUT=FTOUT+2*D39*C17*P8Y1*XNUS**2*KP**2*W4-2*D39*C17*P8Y3*XNUS**2
     X*KP**2*W4-2*D20*P2Y0*C2*XNUS*KP*W2+2*D20*P1Y0*C1+2*D20*C1*P1Y2+4*
     XD20*C2*XNUS*KP*P2Y1*W3-2*D20*C2*XNUS*KP*P2Y2*W2
C                        TERM3
      FTOUT=
     XFTOUT-2*L*W*D39*P1Y0*C6+2*L*W*D39*C6*P1Y2-2*L*W*C43*D4*P1Y0+2*L*
     XW*C43*D4*P1Y2+2*L*W*C42*D4*P1Y0-2*L*W*C42*D4*P1Y2-2*L*W*C43Y*D4*
     XP1Y1+2*L*W*C43Y*D4*P1Y3+2*L*W*P1Y0*C1*D20-2*L*W*C1*D20*P1Y2-2*L*X
     X*C17*D39*XNUS*KP**2*P8Y2*W5+2*L*X*C17*D39*XNUS*KP**2*P8Y0*W5+2*L*
     XC17*D39*XNUS**2*P8Y1*KP**2*W5-2*L*C17*D39*XNUS**2*KP**2*P8Y3*W5-2
     X*L*D39*XNUS*KP*P2Y2*C5*W3+2*L*D39*XNUS*KP*P2Y0*C5*W3-2*L*XNUS*KP*
     XC40*D4*P2Y2*W3+2*L*XNUS*KP*C40*D4*P2Y0*W3+2*L*XNUS*KP*C48*D4*P2Y2
     X*W3-2*L*XNUS*KP*C48*D4*P2Y0*W3+2*L*XNUS*KP*C40Y*D4*P2Y1*W3-2*L*
     XXNUS*KP*C40Y*D4*P2Y3*W3+2*L*XNUS*KP*P2Y2*D20*C2*W3-2*L*XNUS*KP*
     XP2Y0*D20*C2*W3
      FTOUT=FTOUT+2*W*D4*C31*P1Y0-2*W*D4*C31*P1Y2+2*W*D4*C30*P1Y1-2*W*D4
     X*C30*P1Y3-2*W*C11*P1Y0*D20+2*W*C11*D20*P1Y2-2*W*C10*D20*P1Y1+2*W*
     XC10*D20*P1Y3-2*W*C11Y*D20*P1Y1+2*W*C11Y*D20*P1Y3-2*W*C10Y*D20*
     XP1Y2+2*W*C10Y*D20*P1Y4+2*X*C33*XNUS*KP**2*P8Y2*D4*W5-2*X*C33*XNUS
     X*KP**2*D4*P8Y0*W5-2*X*D39*KP*P2Y2*C3*W3+2*X*D39*KP*C3*P2Y0*W3-2*X
     X*XNUS*KP**2*P8Y2*C14*D20*W5+2*X*XNUS*KP**2*P8Y0*C14*D20*W5+2*X*KP
     X*D4*C29*P2Y2*W3-2*X*KP*D4*C29*P2Y0*W3+2*X*KP*D4*C34*P2Y2*W3-2*X*
     XKP*D4*C34*P2Y0*W3+2*X*KP*D4*P2Y2*C25*W3-2*X*KP*D4*P2Y0*C25*W3-2*X
     X*KP*C13*P2Y2*D20*W3+2*X*KP*C13*P2Y0*D20*W3+2*X*KP*C13Y*P2Y1*D20*
     XW3
      FTOUT=FTOUT-2*X*KP*C13Y*P2Y3*D20*W3-2*C33*XNUS**2*P8Y1*KP**2*D4*W5
     X+2*C33*XNUS**2*KP**2*P8Y3*D4*W5+2*XNUS**2*P8Y1*KP**2*C14*D20*W5-2
     X*XNUS**2*KP**2*P8Y3*C14*D20*W5-2*XNUS*KP*D4*C29*P2Y1*W3+2*XNUS*KP
     X*D4*C29*P2Y3*W3-2*XNUS*KP*D4*C34*P2Y1*W3+2*XNUS*KP*D4*C34*P2Y3*W3
     X-2*XNUS*KP*D4*C26*P2Y2*W3+2*XNUS*KP*D4*C26*P2Y0*W3+2*XNUS*KP*D4*
     XP2Y1*C32*W3-2*XNUS*KP*D4*P2Y1*C38*W3-2*XNUS*KP*D4*P2Y3*C32*W3+2*
     XXNUS*KP*D4*P2Y3*C38*W3+2*XNUS*KP*C13*P2Y1*D20*W3-2*XNUS*KP*C13*
     XP2Y3*D20*W3+2*XNUS*KP*C13Y*P2Y2*D20*W3-2*XNUS*KP*C13Y*P2Y4*D20*W3
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X-2*XNUS*KP*C16*P2Y2*D20*W3+2*XNUS*KP*C16*P2Y0*D20*W3+2*XNUS*KP*
XC15*P2Y1*D20*W3
 FTOUT=FTOUT-2*XNUS*KP*C15*P2Y3*D20*W3
C                      TERM4
 FTOUT=
XFTOUT+4*L*W*D20*C53*P1Y1-4*L*W*D20*C41*P1Y1+4*L*W*D20*C43*P1Y2+4*
XL*W*D20*C53Y*P1Y2+4*L*W*D20*C43Y*P1Y3+4*L*W*C21*D39*P1Y2-4*L*X*
XC40Y*D20*P2Y1*W2*KP-4*L*X*C40Y*D20*P2Y2*W3*KP+4*L*X*D20*P2Y1*C44*
XW3*KP-4*L*X*D20*P2Y1*C40*W3*KP+4*L*X*D20*C44*W2*P2Y0*KP-4*L*X*D20
X*C40*W2*P2Y0*KP-4*L*X*C37*P2Y1*D39*W3*KP-4*L*X*C37*D39*W2*P2Y0*KP
X-4*L*C40Y*D20*P2Y2*W2*XNUS*KP-4*L*C40Y*D20*P2Y3*W3*XNUS*KP+4*L*
XD20*P2Y1*C44*W2*XNUS*KP-4*L*D20*P2Y1*C40*W2*XNUS*KP+4*L*D20*P2Y2*
XC44*W3*XNUS*KP-4*L*D20*P2Y2*C40*W3*XNUS*KP+4*L*D20*C53*P1Y0-4*L*
XD20*C41*P1Y0+4*L*D20*C43*P1Y1+4*L*D20*C53Y*P1Y1+4*L*D20*C43Y*P1Y2
X+4*L*C21*D39*P1Y1
  FTOUT=FTOUT-4*L*C37*P2Y1*D39*W2*XNUS*KP-4*L*C37*P2Y2*D39*W3*XNUS*
XKP+8*W*D20*C20*P1Y1+8*W*D20*C20Y*P1Y2-4*W*D20*C27*P1Y2+4*W*D20*
XC30*P1Y1-4*W*D20*C30*P1Y3+4*W*C39*D39*P1Y1+4*W*C39Y*D39*P1Y2+2*X
X**2*D20*P8Y1*C33*W5*KP**2+2*X**2*D20*C33*W4*P8Y0*KP**2+2*X**2*
XP8Y1*C52*D39*W5*KP**2+2*X**2*C52*D39*W4*P8Y0*KP**2+4*X*D20*P8Y1*
XC33*W4*XNUS*KP**2+4*X*D20*P8Y2*C33*W5*XNUS*KP**2-8*X*D20*C22*P2Y1
X*W3*KP-8*X*D20*C22*W2*P2Y0*KP-8*X*D20*C24*P2Y1*W2*KP-8*X*D20*C24*
XP2Y2*W3*KP-4*X*D20*C28*P2Y1*W3*KP-4*X*D20*C28*W2*P2Y0*KP+4*X*D20*
XC35*P2Y1*W3*KP+4*X*D20*C35*W2*P2Y0*KP-4*X*D20*C32*P2Y1*W2*KP-4*X*
XD20*C32*P2Y2*W3*KP
  FTOUT=FTOUT+4*X*D20*P2Y1*C34*W2*KP+4*X*D20*C34*P2Y2*W3*KP+4*X*P8Y1
X*C52*D39*W4*XNUS*KP**2+4*X*P8Y2*C52*D39*W5*XNUS*KP**2-4*X*C45*
XP2Y1*D39*W3*KP-4*X*C45*D39*W2*P2Y0*KP-4*X*C47*P2Y1*D39*W2*KP-4*X*
XC47*P2Y2*D39*W3*KP+2*D20*P8Y2*C33*W4*XNUS**2*KP**2+2*D20*P8Y3*C33
X*W5*XNUS**2*KP**2-2*D20*P7Y1*C33*W5+8*D20*C20*P1Y0+8*D20*C20Y*
XP1Y1-4*D20*C27*P1Y1-4*D20*C30*P1Y2+4*D20*C30*P1Y0-8*D20*C22*P2Y1*
XW2*XNUS*KP-8*D20*C22*P2Y2*W3*XNUS*KP+8*D20*P6Y1*C23*W5-2*D20*P6Y1
X*C33*W5-8*D20*C24*P2Y1*W3*XNUS*KP-8*D20*C24*W2*P2Y0*XNUS*KP-4*D20
X*C28*P2Y1*W2*XNUS*KP-4*D20*C28*P2Y2*W3*XNUS*KP+4*D20*C35*P2Y1*W2*
XXNUS*KP
  FTOUT=FTOUT+4*D20*C35*P2Y2*W3*XNUS*KP-2*D20*C32*P2Y1*W3*XNUS*KP-2*
```

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     XD20*C32*P2Y2*W2*XNUS*KP-2*D20*C32*P2Y3*W3*XNUS*KP-2*D20*C32*W2*
     XP2Y0*XNUS*KP-4*D20*C38*P2Y1*W3*XNUS*KP+4*D20*C38*P2Y2*W2*XNUS*KP+
     X4*D20*C38*P2Y3*W3*XNUS*KP-4*D20*C38*W2*P2Y0*XNUS*KP-4*D20*C29*
     XP2Y1*W3*XNUS*KP+4*D20*C29*P2Y2*W2*XNUS*KP+4*D20*C29*P2Y3*W3*XNUS*
     XKP-4*D20*C29*W2*P2Y0*XNUS*KP+2*D20*P2Y1*C34*W3*XNUS*KP+2*D20*C34*
     XP2Y2*W2*XNUS*KP+2*D20*C34*P2Y3*W3*XNUS*KP+2*D20*C34*W2*P2Y0*XNUS*
     XKP+8*D20*C23*W4*P6Y0-2*D20*C33*W4*P7Y0-2*D20*C33*W4*P6Y0-2*C51*
     XP2Y1*D39*W3*XNUS*KP+2*C51*P2Y2*D39*W2*XNUS*KP+2*C51*P2Y3*D39*W3*
     XXNUS*KP
      FTOUT=FTOUT-2*C51*D39*W2*P2Y0*XNUS*KP+2*P8Y2*C52*D39*W4*XNUS**2*KP
     X**2+2*P8Y3*C52*D39*W5*XNUS**2*KP**2-2*C52*P7Y1*D39*W5+2*C52*P6Y1*
     XD39*W5-2*C52*D39*W4*P7Y0+2*C52*D39*W4*P6Y0+4*C39*D39*P1Y0+4*C39Y*
     XD39*P1Y1+4*P6Y1*C46*D39*W5-4*C45*P2Y1*D39*W2*XNUS*KP-4*C45*P2Y2*
     XD39*W3*XNUS*KP-4*C47*P2Y1*D39*W3*XNUS*KP-4*C47*D39*W2*P2Y0*XNUS*
     XKP+4*C46*D39*W4*P6Y0
C                        TERM5
      FTOUT=
     XFTOUT+8*L*W*D20*C41*P1Y1+2*L*W*D20*C43*P1Y0-2*L*W*D20*C43*P1Y2+2*
     XL*W*D20*C43Y*P1Y1-2*L*W*D20*C43Y*P1Y3+2*L*W*C21*D39*P1Y0-2*L*W*
     XC21*D39*P1Y2-8*L*X*D20*W3*C44*KP*P2Y1-2*L*C40Y*D20*W3*XNUS*KP*
     XP2Y1+2*L*C40Y*D20*W3*XNUS*KP*P2Y3-4*L*D20*W3*C44*XNUS*KP*P2Y2-4*L
     X*D20*W3*C44*XNUS*KP*P2Y0+2*L*D20*W3*C40*XNUS*KP*P2Y2-2*L*D20*W3*
     XC40*XNUS*KP*P2Y0+2*L*W3*C37*D39*XNUS*KP*P2Y2-2*L*W3*C37*D39*XNUS*
     XKP*P2Y0-2*W*D20*C30*P1Y1+2*W*D20*C30*P1Y3-8*W*D20*C20*P1Y1-8*W*
     XD20*C20Y*P1Y2+2*W*D20*C27*P1Y0+6*W*D20*C27*P1Y2+2*W*D20*C31*P1Y0-
     X2*W*D20*C31*P1Y2+4*X**2*D20*W5*C33*P8Y1*KP**2+4*X*D20*W5*C33*P8Y2
     X*XNUS*KP**2
      FTOUT=FTOUT+4*X*D20*W5*C33*XNUS*KP**2*P8Y0+2*X*D20*W3*C32*KP*P2Y2-
     X2*X*D20*W3*C32*KP*P2Y0+8*X*D20*W3*C22*KP*P2Y1+8*X*D20*W3*C24*KP*
     XP2Y2+8*X*D20*W3*C28*KP*P2Y1-2*X*D20*W3*C25*KP*P2Y2+2*X*D20*W3*C25
     X*KP*P2Y0+2*X*D20*W3*C29*KP*P2Y2-2*X*D20*W3*C29*KP*P2Y0-2*X*D20*W3
     X*C34*KP*P2Y2+2*X*D20*W3*C34*KP*P2Y0-2*X*W5*C52*D39*P8Y2*XNUS*KP**
     X2+2*X*W5*C52*D39*XNUS*KP**2*P8Y0-2*X*W3*C51*D39*KP*P2Y2+2*X*W3*
     XC51*D39*KP*P2Y0-8*D20*W5*C23*P6Y1+4*D20*W5*C33*P8Y1*XNUS**2*KP**2
     X-4*D20*W5*C33*P7Y1+4*D20*W5*C33*P6Y1+2*D20*W3*C38*XNUS*KP*P2Y1-2*
     XD20*W3*C38*XNUS*KP*P2Y3+8*D20*W3*C22*XNUS*KP*P2Y2+2*D20*W3*C26*
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     XXNUS*KP*P2Y2
      FTOUT=FTOUT-2*D20*W3*C26*XNUS*KP*P2Y0+8*D20*W3*C24*XNUS*KP*P2Y1+6*
     XD20*W3*C28*XNUS*KP*P2Y2+2*D20*W3*C28*XNUS*KP*P2Y0-2*D20*W3*C35*
     XXNUS*KP*P2Y2+2*D20*W3*C35*XNUS*KP*P2Y0+2*D20*W3*C29*XNUS*KP*P2Y1-
     X2*D20*W3*C29*XNUS*KP*P2Y3+2*W5*C52*D39*P8Y1*XNUS**2*KP**2-2*W5*
     XC52*D39*P8Y3*XNUS**2*KP**2-2*W3*C50*D39*XNUS*KP*P2Y2+2*W3*C50*D39
     X*XNUS*KP*P2Y0+2*W3*C51*D39*XNUS*KP*P2Y1-2*W3*C51*D39*XNUS*KP*P2Y3
C                        TERM6
      FTOUT=
     XFTOUT+4*L*W*D20*C20*P1Y0+4*L*W*D20*C20Y*P1Y1+2*L*W*D20*C30*P1Y0-2
     X*L*W*D20*C30*P1Y2+4*L*W*C39*D39*P1Y0+4*L*W*C39Y*D39*P1Y1+4*L*X**2
     X*C33*D20*KP**2*P8Y1*W4+4*L*X**2*C33*D20*KP**2*P8Y0*W5+2*L*X**2*KP
     X**2*C52*D39*P8Y1*W4+2*L*X**2*KP**2*C52*D39*P8Y0*W5-4*L*X*W3*D20*
     XC22*KP*P2Y0-4*L*X*W3*D20*C24*KP*P2Y1+4*L*X*W3*D20*KP*C35*P2Y0+4*L
     X*X*W3*D20*KP*C34*P2Y1-4*L*X*W3*D20*KP*C32*P2Y1-4*L*X*W3*C45*KP*
     XD39*P2Y0-4*L*X*W3*C47*KP*D39*P2Y1+8*L*X*C33*D20*XNUS*KP**2*P8Y1*
     XW5+4*L*X*C33*D20*XNUS*KP**2*P8Y2*W4+4*L*X*C33*D20*XNUS*KP**2*P8Y0
     X*W4-4*L*X*D20*W2*C22*KP*P2Y1-4*L*X*D20*W2*C24*KP*P2Y2+2*L*X*D20*
     XW2*KP*C29*P2Y0
      FTOUT=FTOUT-2*L*X*D20*W2*KP*C29*P2Y2+4*L*X*D20*W2*KP*C35*P2Y1+2*L*
     XX*D20*W2*KP*C34*P2Y0+2*L*X*D20*W2*KP*C34*P2Y2+2*L*X*D20*W2*KP*C25
     X*P2Y0-2*L*X*D20*W2*KP*C25*P2Y2-2*L*X*D20*W2*KP*C32*P2Y0-2*L*X*D20
     X*W2*KP*C32*P2Y2-4*L*X*W2*C45*KP*D39*P2Y1-4*L*X*W2*C47*KP*D39*P2Y2
     X+2*L*X*W2*C51*KP*D39*P2Y0-2*L*X*W2*C51*KP*D39*P2Y2+4*L*X*XNUS*KP
     X**2*C52*D39*P8Y1*W5+2*L*X*XNUS*KP**2*C52*D39*P8Y2*W4+2*L*X*XNUS*
     XKP**2*C52*D39*P8Y0*W4-4*L*W3*D20*C22*XNUS*KP*P2Y1-4*L*W3*D20*C24*
     XXNUS*KP*P2Y0-2*L*W3*D20*XNUS*KP*C29*P2Y0+2*L*W3*D20*XNUS*KP*C29*
     XP2Y2+4*L*W3*D20*XNUS*KP*C35*P2Y1+4*L*W3*D20*XNUS*KP*C34*P2Y2-4*L*
     XW3*D20*XNUS*KP*C32*P2Y2
      FTOUT=FTOUT-2*L*W3*D20*XNUS*KP*C38*P2Y0+2*L*W3*D20*XNUS*KP*C38*
     XP2Y2-4*L*W3*C45*XNUS*KP*D39*P2Y1-4*L*W3*C47*XNUS*KP*D39*P2Y0-2*L*
     XW3*XNUS*C51*KP*D39*P2Y0+2*L*W3*XNUS*C51*KP*D39*P2Y2+4*L*C33*D20*
     XXNUS**2*KP**2*P8Y1*W4+4*L*C33*D20*XNUS**2*KP**2*P8Y2*W5-4*L*C33*
     XD20*P7Y1*W4-4*L*C33*D20*P7Y0*W5-4*L*D20*W2*C22*XNUS*KP*P2Y2-4*L*
     XD20*W2*C24*XNUS*KP*P2Y1+2*L*D20*W2*XNUS*C28*KP*P2Y0-2*L*D20*W2*
     XXNUS*C28*KP*P2Y2+2*L*D20*W2*XNUS*KP*C35*P2Y0+2*L*D20*W2*XNUS*KP*
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XC35*P2Y2-2*L*D20*W2*XNUS*KP*C26*P2Y0+2*L*D20*W2*XNUS*KP*C26*P2Y2+
X4*L*D20*W2*XNUS*KP*C34*P2Y1-4*L*D20*W2*XNUS*KP*C32*P2Y1+4*L*D20*
XC20*P1Y1
 FTOUT=FTOUT+4*L*D20*C20Y*P1Y2+2*L*D20*C27*P1Y0-2*L*D20*C27*P1Y2-2*
XL*D20*C31*P1Y0+2*L*D20*C31*P1Y2+4*L*D20*C23*P6Y1*W4+4*L*D20*C23*
XP6Y0*W5+2*L*W2*C50*XNUS*KP*D39*P2Y0-2*L*W2*C50*XNUS*KP*D39*P2Y2-4
X*L*W2*C45*XNUS*KP*D39*P2Y2-4*L*W2*C47*XNUS*KP*D39*P2Y1+4*L*C39*
XD39*P1Y1+4*L*C39Y*D39*P1Y2+2*L*XNUS**2*KP**2*C52*D39*P8Y1*W4+2*L*
XXNUS**2*KP**2*C52*D39*P8Y2*W5-2*L*C52*D39*P7Y1*W4+2*L*C52*D39*
XP6Y1*W4-2*L*C52*D39*P7Y0*W5+2*L*C52*D39*P6Y0*W5+4*L*C46*D39*P6Y1*
XW4+4*L*C46*D39*P6Y0*W5+4*W*D20*C53*P1Y0+4*W*D20*C43*P1Y1+4*W*D20*
XC53Y*P1Y1+4*W*D20*C43Y*P1Y2+4*W*C21*D39*P1Y1-4*X*W3*D20*C40*KP*
XP2Y0
 FTOUT=FTOUT-4*X*W3*D20*C40Y*KP*P2Y1-4*X*W3*KP*C37*D39*P2Y0-4*X*D20
X*W2*C40*KP*P2Y1-4*X*D20*W2*C40Y*KP*P2Y2-4*X*W2*KP*C37*D39*P2Y1-4*
XW3*D20*C40*XNUS*KP*P2Y1-4*W3*D20*C40Y*XNUS*KP*P2Y2-4*W3*XNUS*KP*
XC37*D39*P2Y1+4*D20*W2*C48*XNUS*KP*P2Y0-4*D20*W2*C48*XNUS*KP*P2Y2-
X2*D20*W2*C40*XNUS*KP*P2Y0-2*D20*W2*C40*XNUS*KP*P2Y2-2*D20*W2*C40Y
X*XNUS*KP*P2Y1-2*D20*W2*C40Y*XNUS*KP*P2Y3+4*D20*C53*P1Y1-4*D20*C42
X*P1Y0+4*D20*C42*P1Y2+2*D20*C43*P1Y0+2*D20*C43*P1Y2+4*D20*C53Y*
XP1Y2+2*D20*C43Y*P1Y1+2*D20*C43Y*P1Y3-2*W2*XNUS*KP*C37*D39*P2Y0-2*
XW2*XNUS*KP*C37*D39*P2Y2+2*C21*D39*P1Y0+2*C21*D39*P1Y2
C                             TERM1
  FTOUT=
XFTOUT+2*L*W*C1*D11*P1Y1-2*L*W*C1*D11*P1Y3+2*L*W*C1*D11Y*P1Y2-2*L*
XW*C1*D11Y*P1Y4+2*L*X*D10*C2*W3*KP*P2Y1-2*L*X*D10*C2*W3*KP*P2Y3-2*
XL*X*D10*C2*W2*P2Y0*KP+2*L*X*D10*C2*W2*KP*P2Y2+2*L*X*C2*D10Y*W3*KP
X*P2Y2-2*L*X*C2*D10Y*W3*KP*P2Y4-2*L*X*C2*D10Y*W2*KP*P2Y1+2*L*X*C2*
XD10Y*W2*KP*P2Y3-2*L*C1*D11*P1Y0+2*L*C1*D11*P1Y2-2*L*C1*D11Y*P1Y1+
X2*L*C1*D11Y*P1Y3-2*L*C2*D11*W3*XNUS*KP*P2Y1+2*L*C2*D11*W3*XNUS*KP
X*P2Y3+2*L*C2*D11*W2*P2Y0*XNUS*KP-2*L*C2*D11*W2*XNUS*KP*P2Y2-2*L*
XC2*D11Y*W3*XNUS*KP*P2Y2+2*L*C2*D11Y*W3*XNUS*KP*P2Y4+2*L*C2*D11Y*
XW2*XNUS*KP*P2Y1-2*L*C2*D11Y*W2*XNUS*KP*P2Y3-2*W*C4*D10*P1Y1+2*W*
XC4*D10*P1Y3
  FTOUT=FTOUT-2*W*C4*D10Y*P1Y2+2*W*C4*D10Y*P1Y4-2*W*C11*D10*P1Y2+2*W
X*C11*D10*P1Y4-2*W*C11*D11*P1Y1+2*W*C11*D11*P1Y3-2*W*C11*D10Y*P1Y3
```

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X+2*W*C11*D10Y*P1Y5-2*W*C11*D11Y*P1Y2+2*W*C11*D11Y*P1Y4-2*W*C10*
XD10*P1Y1+2*W*C10*D10*P1Y3-2*W*C10*D11*P1Y2+2*W*C10*D11*P1Y4-2*W*
XC10*D10Y*P1Y2+2*W*C10*D10Y*P1Y4-2*W*C10*D11Y*P1Y3+2*W*C10*D11Y*
XP1Y5-2*W*C11Y*D10*P1Y3+2*W*C11Y*D10*P1Y5-2*W*C11Y*D11*P1Y2+2*W*
XC11Y*D11*P1Y4-2*W*C11Y*D10Y*P1Y4+2*W*C11Y*D10Y*P1Y6-2*W*C11Y*D11Y
X*P1Y3+2*W*C11Y*D11Y*P1Y5-2*W*C10Y*D10*P1Y2+2*W*C10Y*D10*P1Y4-2*W*
XC10Y*D11*P1Y3+2*W*C10Y*D11*P1Y5-2*W*C10Y*D10Y*P1Y3+2*W*C10Y*D10Y*
XP1Y5
 FTOUT=FTOUT-2*W*C10Y*D11Y*P1Y4+2*W*C10Y*D11Y*P1Y6-4*X**3*C17*D5*
XP5Y1*W7*KP**3-2*X**3*C17*D5*W7*P8Y1*KP**3+4*X**3*C17*D5*W7*KP**3*
XP2Y1+2*X**3*C17*D5*W6*P8Y0*KP**3+4*X**3*C17*D5*W6*P5Y0*KP**3-4*X
X**3*C17*D5*W6*P2Y0*KP**3-4*X**2*C17*D5*P5Y1*W7*XNUS*KP**3+8*X**2*
XC17*D5*P5Y1*W6*XNUS*KP**3-8*X**2*C17*D5*P5Y2*W7*XNUS*KP**3-4*X**2
X*C17*D5*W7*P8Y1*XNUS*KP**3-2*X**2*C17*D5*W7*P8Y2*XNUS*KP**3+8*X**
X2*C17*D5*W7*XNUS*KP**3*P2Y1+4*X**2*C17*D5*W7*XNUS*KP**3*P2Y2+4*X
X**2*C17*D5*W6*P8Y0*XNUS*KP**3+4*X**2*C17*D5*W6*P5Y0*XNUS*KP**3-8*
XX**2*C17*D5*W6*P2Y0*XNUS*KP**3+2*X**2*C17*D5*W6*P8Y1*XNUS*KP**3-4
X*X**2*C17*D5*W6*XNUS*KP**3*P2Y1
 FTOUT=FTOUT-2*X**2*D5*P5Y1*C5*W5*KP**2+2*X**2*D5*C5*W4*P5Y0*KP**2+
X2*X*C17*D5*P5Y1*W7*KP**3+4*X*C17*D5*P5Y1*W6*XNUS**2*KP**3-4*X*C17
X*D5*P5Y2*W7*XNUS**2*KP**3+8*X*C17*D5*P5Y2*W6*XNUS**2*KP**3-8*X*
XC17*D5*P5Y3*W7*XNUS**2*KP**3-840*X*C17*D5*P0*W6*KP+2*X*C17*D5*W7*
XP8Y1*KP**3-4*X*C17*D5*W7*P8Y2*XNUS**2*KP**3-2*X*C17*D5*W7*P8Y3*
XXNUS**2*KP**3+2*X*C17*D5*W7*P4Y1*KP+8*X*C17*D5*W7*XNUS**2*KP**3*
XP2Y2+4*X*C17*D5*W7*XNUS**2*KP**3*P2Y3-4*X*C17*D5*W7*KP**3*P2Y1-2*
XX*C17*D5*W7*KP*P3Y1-2*X*C17*D5*W6*P8Y0*KP**3-2*X*C17*D5*W6*P5Y0*
XKP**3-2*X*C17*D5*W6*P4Y0*KP+2*X*C17*D5*W6*P3Y0*KP+4*X*C17*D5*W6*
XP2Y0*KP**3
 FTOUT=FTOUT+4*X*C17*D5*W6*P8Y1*XNUS**2*KP**3+2*X*C17*D5*W6*P8Y2*
XXNUS**2*KP**3-8*X*C17*D5*W6*XNUS**2*KP**3*P2Y1-4*X*C17*D5*W6*XNUS
X**2*KP**3*P2Y2+4*X*D5*P5Y1*C5*W4*XNUS*KP**2-4*X*D5*P5Y2*C5*W5*
XXNUS*KP**2+2*X*C13*D10*W3*KP*P2Y2-2*X*C13*D10*W3*KP*P2Y4-2*X*C13*
XD10*W2*KP*P2Y1+2*X*C13*D10*W2*KP*P2Y3+2*X*C13*D11*W3*KP*P2Y1-2*X*
XC13*D11*W3*KP*P2Y3-2*X*C13*D11*W2*P2Y0*KP+2*X*C13*D11*W2*KP*P2Y2+
X2*X*C13*D10Y*W3*KP*P2Y3-2*X*C13*D10Y*W3*KP*P2Y5-2*X*C13*D10Y*W2*
XKP*P2Y2+2*X*C13*D10Y*W2*KP*P2Y4+2*X*C13*D11Y*W3*KP*P2Y2-2*X*C13*
```

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XD11Y*W3*KP*P2Y4-2*X*C13*D11Y*W2*KP*P2Y1+2*X*C13*D11Y*W2*KP*P2Y3+2
X*X*C7*D10*W3*KP*P2Y1
 FTOUT=FTOUT-2*X*C7*D10*W3*KP*P2Y3-2*X*C7*D10*W2*P2Y0*KP+2*X*C7*D10
X*W2*KP*P2Y2+2*X*C7*D10Y*W3*KP*P2Y2-2*X*C7*D10Y*W3*KP*P2Y4-2*X*C7*
XD10Y*W2*KP*P2Y1+2*X*C7*D10Y*W2*KP*P2Y3+2*X*C9*D10*W3*KP*P2Y2-2*X*
XC9*D10*W3*KP*P2Y4-2*X*C9*D10*W2*KP*P2Y1+2*X*C9*D10*W2*KP*P2Y3+2*X
X*C9*D10Y*W3*KP*P2Y3-2*X*C9*D10Y*W3*KP*P2Y5-2*X*C9*D10Y*W2*KP*P2Y2
X+2*X*C9*D10Y*W2*KP*P2Y4+2*X*C13Y*D10*W3*KP*P2Y3-2*X*C13Y*D10*W3*
XKP*P2Y5-2*X*C13Y*D10*W2*KP*P2Y2+2*X*C13Y*D10*W2*KP*P2Y4+2*X*C13Y*
XD11*W3*KP*P2Y2-2*X*C13Y*D11*W3*KP*P2Y4-2*X*C13Y*D11*W2*KP*P2Y1+2*
XX*C13Y*D11*W2*KP*P2Y3+2*X*C13Y*D10Y*W3*KP*P2Y4-2*X*C13Y*D10Y*W3*
XKP*P2Y6
 FTOUT=FTOUT-2*X*C13Y*D10Y*W2*KP*P2Y3+2*X*C13Y*D10Y*W2*KP*P2Y5+2*X*
XC13Y*D11Y*W3*KP*P2Y3-2*X*C13Y*D11Y*W3*KP*P2Y5-2*X*C13Y*D11Y*W2*KP
X*P2Y2+2*X*C13Y*D11Y*W2*KP*P2Y4+2*X*D10*C14*W5*P8Y2*XNUS*KP**2-2*X
X*D10*C14*W5*P8Y4*XNUS*KP**2-2*X*D10*C14*W4*P8Y1*XNUS*KP**2+2*X*
XD10*C14*W4*P8Y3*XNUS*KP**2+2*X*D11*C14*W5*P8Y1*XNUS*KP**2-2*X*D11
X*C14*W5*P8Y3*XNUS*KP**2-2*X*D11*C14*W4*P8Y0*XNUS*KP**2+2*X*D11*
XC14*W4*P8Y2*XNUS*KP**2+2*X*D10Y*C14*W5*P8Y3*XNUS*KP**2-2*X*D10Y*
XC14*W5*P8Y5*XNUS*KP**2-2*X*D10Y*C14*W4*P8Y2*XNUS*KP**2+2*X*D10Y*
XC14*W4*P8Y4*XNUS*KP**2+2*X*D11Y*C14*W5*P8Y2*XNUS*KP**2-2*X*D11Y*
XC14*W5*P8Y4*XNUS*KP**2
 FTOUT=FTOUT-2*X*D11Y*C14*W4*P8Y1*XNUS*KP**2+2*X*D11Y*C14*W4*P8Y3*
XXNUS*KP**2-2*C17*D5*P5Y1*W6*XNUS*KP**3+2*C17*D5*P5Y2*W7*XNUS*KP**
X3+4*C17*D5*P5Y3*W6*XNUS**3*KP**3-4*C17*D5*P5Y4*W7*XNUS**3*KP**3+
X280*C17*D5*P0*W7*XNUS*KP+2*C17*D5*W7*P8Y2*XNUS*KP**3-2*C17*D5*W7*
XP8Y4*XNUS**3*KP**3+2*C17*D5*W7*P4Y2*XNUS*KP+4*C17*D5*W7*XNUS**3*
XKP**3*P2Y4-4*C17*D5*W7*XNUS*KP**3*P2Y2-2*C17*D5*W7*XNUS*KP*P3Y2-2
X*C17*D5*W6*P8Y1*XNUS*KP**3+2*C17*D5*W6*P8Y3*XNUS**3*KP**3-2*C17*
XD5*W6*P4Y1*XNUS*KP-4*C17*D5*W6*XNUS**3*KP**3*P2Y3+4*C17*D5*W6*
XXNUS*KP**3*P2Y1+2*C17*D5*W6*XNUS*KP*P3Y1+2*D5*C6*W3*XNUS*KP*P2Y1-
X2*D5*C6*W3*XNUS*KP*P2Y3
 FTOUT=FTOUT-2*D5*C6*W2*P2Y0*XNUS*KP+2*D5*C6*W2*XNUS*KP*P2Y2+2*D5*
XP5Y2*C5*W4*XNUS**2*KP**2-2*D5*P5Y3*C5*W5*XNUS**2*KP**2+2*D5*C5*W5
X*P4Y1-2*D5*C5*W5*P3Y1-2*D5*C5*W4*P4Y0+2*D5*C5*W4*P3Y0-2*P7Y1*D10*
XC14*W5+2*P7Y1*D10Y*C14*W4-2*P7Y2*D10*C14*W4-2*P7Y2*D10Y*C14*W5+2*
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XP7Y3*D10*C14*W5-2*P7Y3*D10Y*C14*W4+2*P7Y4*D10Y*C14*W5+2*C4*D10*
XP1Y0-2*C4*D10*P1Y2+2*C4*D10Y*P1Y1-2*C4*D10Y*P1Y3+2*C11*D10*P1Y1-2
X*C11*D10*P1Y3+2*C11*D11*P1Y0-2*C11*D11*P1Y2+2*C11*D10Y*P1Y2-2*C11
X*D10Y*P1Y4+2*C11*D11Y*P1Y1-2*C11*D11Y*P1Y3+2*C13*D10*W3*XNUS*KP*
XP2Y1-2*C13*D10*W3*XNUS*KP*P2Y3-2*C13*D10*W2*P2Y0*XNUS*KP+2*C13*
XD10*W2*XNUS*KP*P2Y2
 FTOUT=FTOUT+2*C13*D11*W3*XNUS*KP*P2Y2-2*C13*D11*W3*XNUS*KP*P2Y4-2*
XC13*D11*W2*XNUS*KP*P2Y1+2*C13*D11*W2*XNUS*KP*P2Y3+2*C13*D10Y*W3*
XXNUS*KP*P2Y2-2*C13*D10Y*W3*XNUS*KP*P2Y4-2*C13*D10Y*W2*XNUS*KP*
XP2Y1+2*C13*D10Y*W2*XNUS*KP*P2Y3+2*C13*D11Y*W3*XNUS*KP*P2Y3-2*C13*
XD11Y*W3*XNUS*KP*P2Y5-2*C13*D11Y*W2*XNUS*KP*P2Y2+2*C13*D11Y*W2*
XXNUS*KP*P2Y4+2*C10*D10*P1Y0-2*C10*D10*P1Y2+2*C10*D11*P1Y1-2*C10*
XD11*P1Y3+2*C10*D10Y*P1Y1-2*C10*D10Y*P1Y3+2*C10*D11Y*P1Y2-2*C10*
XD11Y*P1Y4+2*C15*D10*W3*XNUS*KP*P2Y1-2*C15*D10*W3*XNUS*KP*P2Y3-2*
XC15*D10*W2*P2Y0*XNUS*KP+2*C15*D10*W2*XNUS*KP*P2Y2+2*C15*D11*W3*
XXNUS*KP*P2Y2
 FTOUT=FTOUT-2*C15*D11*W3*XNUS*KP*P2Y4-2*C15*D11*W2*XNUS*KP*P2Y1+2*
XC15*D11*W2*XNUS*KP*P2Y3+2*C15*D10Y*W3*XNUS*KP*P2Y2-2*C15*D10Y*W3*
XXNUS*KP*P2Y4-2*C15*D10Y*W2*XNUS*KP*P2Y1+2*C15*D10Y*W2*XNUS*KP*
XP2Y3+2*C15*D11Y*W3*XNUS*KP*P2Y3-2*C15*D11Y*W3*XNUS*KP*P2Y5-2*C15*
XD11Y*W2*XNUS*KP*P2Y2+2*C15*D11Y*W2*XNUS*KP*P2Y4+2*C16*D10*W3*XNUS
X*KP*P2Y2-2*C16*D10*W3*XNUS*KP*P2Y4-2*C16*D10*W2*XNUS*KP*P2Y1+2*
XC16*D10*W2*XNUS*KP*P2Y3+2*C16*D11*W3*XNUS*KP*P2Y1-2*C16*D11*W3*
XXNUS*KP*P2Y3-2*C16*D11*W2*P2Y0*XNUS*KP+2*C16*D11*W2*XNUS*KP*P2Y2+
X2*C16*D10Y*W3*XNUS*KP*P2Y3-2*C16*D10Y*W3*XNUS*KP*P2Y5-2*C16*D10Y*
XW2*XNUS*KP*P2Y2
 FTOUT=FTOUT+2*C16*D10Y*W2*XNUS*KP*P2Y4+2*C16*D11Y*W3*XNUS*KP*P2Y2-
X2*C16*D11Y*W3*XNUS*KP*P2Y4-2*C16*D11Y*W2*XNUS*KP*P2Y1+2*C16*D11Y*
XW2*XNUS*KP*P2Y3+2*C11Y*D10*P1Y2-2*C11Y*D10*P1Y4+2*C11Y*D11*P1Y1-2
X*C11Y*D11*P1Y3+2*C11Y*D10Y*P1Y3-2*C11Y*D10Y*P1Y5+2*C11Y*D11Y*P1Y2
X-2*C11Y*D11Y*P1Y4+2*C7*D10*W3*XNUS*KP*P2Y2-2*C7*D10*W3*XNUS*KP*
XP2Y4-2*C7*D10*W2*XNUS*KP*P2Y1+2*C7*D10*W2*XNUS*KP*P2Y3+2*C7*D10Y*
XW3*XNUS*KP*P2Y3-2*C7*D10Y*W3*XNUS*KP*P2Y5-2*C7*D10Y*W2*XNUS*KP*
XP2Y2+2*C7*D10Y*W2*XNUS*KP*P2Y4+2*C10Y*D10*P1Y1-2*C10Y*D10*P1Y3+2*
XC10Y*D11*P1Y2-2*C10Y*D11*P1Y4+2*C10Y*D10Y*P1Y2-2*C10Y*D10Y*P1Y4+2
X*C10Y*D11Y*P1Y3
```

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      FTOUT=FTOUT-2*C10Y*D11Y*P1Y5+2*C9*D10*W3*XNUS*KP*P2Y1-2*C9*D10*W3*
     XXNUS*KP*P2Y3-2*C9*D10*W2*P2Y0*XNUS*KP+2*C9*D10*W2*XNUS*KP*P2Y2+2*
     XC9*D10Y*W3*XNUS*KP*P2Y2-2*C9*D10Y*W3*XNUS*KP*P2Y4-2*C9*D10Y*W2*
     XXNUS*KP*P2Y1+2*C9*D10Y*W2*XNUS*KP*P2Y3+2*C13Y*D10*W3*XNUS*KP*P2Y2
     X-2*C13Y*D10*W3*XNUS*KP*P2Y4-2*C13Y*D10*W2*XNUS*KP*P2Y1+2*C13Y*D10
     X*W2*XNUS*KP*P2Y3+2*C13Y*D11*W3*XNUS*KP*P2Y3-2*C13Y*D11*W3*XNUS*KP
     X*P2Y5-2*C13Y*D11*W2*XNUS*KP*P2Y2+2*C13Y*D11*W2*XNUS*KP*P2Y4+2*
     XC13Y*D10Y*W3*XNUS*KP*P2Y3-2*C13Y*D10Y*W3*XNUS*KP*P2Y5-2*C13Y*D10Y
     X*W2*XNUS*KP*P2Y2+2*C13Y*D10Y*W2*XNUS*KP*P2Y4+2*C13Y*D11Y*W3*XNUS*
     XKP*P2Y4
      FTOUT=FTOUT-2*C13Y*D11Y*W3*XNUS*KP*P2Y6-2*C13Y*D11Y*W2*XNUS*KP*
     XP2Y3+2*C13Y*D11Y*W2*XNUS*KP*P2Y5+2*D10*C14*W5*P8Y1*XNUS**2*KP**2-
     X2*D10*C14*W5*P8Y3*XNUS**2*KP**2-2*D10*C14*W4*P8Y0*XNUS**2*KP**2+2
     X*D10*C14*W4*P7Y0+2*D10*C14*W4*P8Y2*XNUS**2*KP**2+2*D11*C14*W5*
     XP8Y2*XNUS**2*KP**2-2*D11*C14*W5*P8Y4*XNUS**2*KP**2-2*D11*C14*W4*
     XP8Y1*XNUS**2*KP**2+2*D11*C14*W4*P8Y3*XNUS**2*KP**2+2*D10Y*C14*W5*
     XP8Y2*XNUS**2*KP**2-2*D10Y*C14*W5*P8Y4*XNUS**2*KP**2-2*D10Y*C14*W4
     X*P8Y1*XNUS**2*KP**2+2*D10Y*C14*W4*P8Y3*XNUS**2*KP**2+2*D11Y*C14*
     XW5*P8Y3*XNUS**2*KP**2-2*D11Y*C14*W5*P8Y5*XNUS**2*KP**2-2*D11Y*C14
     X*W4*P8Y2*XNUS**2*KP**2
      FTOUT=FTOUT+2*D11Y*C14*W4*P8Y4*XNUS**2*KP**2
C                         TERM2
      FTOUT=
     XFTOUT-2*L*X**2*C2*D48*KP**2*W5*P5Y0+2*L*X**2*C2*D48*KP**2*W4*
     XP5Y1-4*L*X*C2*D48*XNUS*KP**2*W5*P5Y1+2*L*X*C2*D48*XNUS*KP**2*W4*
     XP5Y2+2*L*X*C2*D48*XNUS*KP**2*W4*P5Y0-2*L*X*C2*KP*W2*D43*P2Y2+2*L*
     XX*C2*KP*W2*D43*P2Y0+2*L*X*C2*KP*W2*D43Y*P2Y1-2*L*X*C2*KP*W2*D43Y*
     XP2Y3-2*L*C2*D48*XNUS**2*KP**2*W5*P5Y2+2*L*C2*D48*XNUS**2*KP**2*W4
     X*P5Y1+2*L*C2*D48*W5*P4Y0-2*L*C2*D48*W5*P3Y0-2*L*C2*D48*W4*P4Y1+2*
     XL*C2*D48*W4*P3Y1+2*L*C2*XNUS*KP*D53*W2*P2Y2-2*L*C2*XNUS*KP*D53*W2
     X*P2Y0-2*L*C2*XNUS*KP*W2*D53Y*P2Y1+2*L*C2*XNUS*KP*W2*D53Y*P2Y3-2*L
     X*D48*XNUS*KP*W3*C1*P2Y2+2*L*D48*XNUS*KP*W3*C1*P2Y0+2*L*P1Y1*C1*
     XD53Y
      FTOUT=FTOUT-2*L*P1Y2*D53*C1-2*L*P1Y3*C1*D53Y+2*L*D53*C1*P1Y0-2*X**
     X3*D48*KP**3*P8Y1*C14*W6+2*X**3*D48*KP**3*C14*W7*P8Y0+4*X**3*D48*
     XKP**3*C14*W7*P5Y0-4*X**3*D48*KP**3*C14*W7*P2Y0-4*X**3*D48*KP**3*
```

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XC14*W6*P5Y1+4*X**3*D48*KP**3*C14*W6*P2Y1+2*X**2*D48*XNUS*KP**3*
XP8Y1*C14*W7-4*X**2*D48*XNUS*KP**3*P8Y1*C14*W6+8*X**2*D48*XNUS*KP
X**3*C14*W7*P5Y1-4*X**2*D48*XNUS*KP**3*C14*W7*P2Y1+4*X**2*D48*XNUS
X*KP**3*C14*W7*P8Y0+4*X**2*D48*XNUS*KP**3*C14*W7*P5Y0-8*X**2*D48*
XXNUS*KP**3*C14*W7*P2Y0-4*X**2*D48*XNUS*KP**3*C14*W6*P5Y1-4*X**2*
XD48*XNUS*KP**3*C14*W6*P5Y2+8*X**2*D48*XNUS*KP**3*C14*W6*P2Y1-2*X
X**2*D48*XNUS*KP**3*C14*W6*P8Y0
 FTOUT=FTOUT-4*X**2*D48*XNUS*KP**3*C14*W6*P5Y0+4*X**2*D48*XNUS*KP**
X3*C14*W6*P2Y0+2*X**2*D48*KP**2*C15*W5*P5Y1-2*X**2*D48*KP**2*C15*
XW4*P5Y0+2*X**2*D48*KP**2*C16*W5*P5Y0-2*X**2*D48*KP**2*C16*W4*P5Y1
X+4*X*D48*XNUS**2*KP**3*P8Y1*C14*W7+2*X*D48*XNUS**2*KP**3*P8Y2*C14
X*W7-2*X*D48*XNUS**2*KP**3*P8Y3*C14*W6+4*X*D48*XNUS**2*KP**3*C14*
XW7*P5Y1+8*X*D48*XNUS**2*KP**3*C14*W7*P5Y2-8*X*D48*XNUS**2*KP**3*
XC14*W7*P2Y1-4*X*D48*XNUS**2*KP**3*C14*W7*P2Y2-4*X*D48*XNUS**2*KP
X**3*C14*W6*P5Y1-4*X*D48*XNUS**2*KP**3*C14*W6*P5Y3+4*X*D48*XNUS**2
X*KP**3*C14*W6*P2Y3-4*X*D48*XNUS**2*KP**3*C14*W6*P8Y0-4*X*D48*XNUS
X**2*KP**3*C14*W6*P5Y0
  FTOUT=FTOUT+8*X*D48*XNUS**2*KP**3*C14*W6*P2Y0-2*X*D48*XNUS*KP**2*
XC7*W4*P5Y2+2*X*D48*XNUS*KP**2*C7*W4*P5Y0+2*X*D48*XNUS*KP**2*C9*W4
X*P5Y1-2*X*D48*XNUS*KP**2*C9*W4*P5Y3+4*X*D48*XNUS*KP**2*C15*W5*
XP5Y2-4*X*D48*XNUS*KP**2*C15*W4*P5Y1+4*X*D48*XNUS*KP**2*C16*W5*
XP5Y1-4*X*D48*XNUS*KP**2*C16*W4*P5Y2+2*X*D48*XNUS*KP**2*C13*W5*
XP5Y2-2*X*D48*XNUS*KP**2*C13*W5*P5Y0+2*X*D48*XNUS*KP**2*C13*W4*
XP5Y1-2*X*D48*XNUS*KP**2*C13*W4*P5Y3-2*X*D48*XNUS*KP**2*C13Y*W5*
XP5Y1+2*X*D48*XNUS*KP**2*C13Y*W5*P5Y3+2*X*D48*XNUS*KP**2*C13Y*W4*
XP5Y2-2*X*D48*XNUS*KP**2*C13Y*W4*P5Y4+2*X*D48*KP**3*P8Y1*C14*W6-2*
XX*D48*KP**3*C14*W7*P8Y0
  FTOUT=FTOUT-2*X*D48*KP**3*C14*W7*P5Y0+4*X*D48*KP**3*C14*W7*P2Y0+2*
XX*D48*KP**3*C14*W6*P5Y1-4*X*D48*KP**3*C14*W6*P2Y1-840*X*D48*KP*P0
X*C14*W7-2*X*D48*KP*C14*W7*P4Y0+2*X*D48*KP*C14*W7*P3Y0+2*X*D48*KP*
XC14*W6*P4Y1-2*X*D48*KP*C14*W6*P3Y1+2*X*XNUS*KP**2*P8Y1*C14*W4*D43
X+2*X*XNUS*KP**2*P8Y1*C14*W4*D53Y-2*X*XNUS*KP**2*P8Y2*C14*W4*D53+2
X*X*XNUS*KP**2*P8Y2*C14*W4*D43Y-2*X*XNUS*KP**2*P8Y3*C14*W4*D43-2*X
X*XNUS*KP**2*P8Y3*C14*W4*D53Y-2*X*XNUS*KP**2*C14*P8Y4*W4*D43Y+2*X*
XXNUS*KP**2*C14*W4*D53*P8Y0-2*X*KP*C7*W2*D43*P2Y2+2*X*KP*C7*W2*D43
X*P2Y0+2*X*KP*C7*W2*D43Y*P2Y1-2*X*KP*C7*W2*D43Y*P2Y3+2*X*KP*C9*W2*
```

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XD43*P2Y1
 FTOUT=FTOUT-2*X*KP*C9*W2*D43*P2Y3+2*X*KP*C9*W2*D43Y*P2Y2-2*X*KP*C9
X*W2*D43Y*P2Y4-2*X*KP*C13*D53*W2*P2Y2+2*X*KP*C13*D53*W2*P2Y0+2*X*
XKP*C13*W2*D43*P2Y1-2*X*KP*C13*W2*D43*P2Y3+2*X*KP*C13*W2*D53Y*P2Y1
X-2*X*KP*C13*W2*D53Y*P2Y3+2*X*KP*C13*W2*D43Y*P2Y2-2*X*KP*C13*W2*
XD43Y*P2Y4+2*X*KP*C13Y*D53*W2*P2Y1-2*X*KP*C13Y*D53*W2*P2Y3+2*X*KP*
XC13Y*W2*D43*P2Y2-2*X*KP*C13Y*W2*D43*P2Y4+2*X*KP*C13Y*W2*D53Y*P2Y2
X-2*X*KP*C13Y*W2*D53Y*P2Y4+2*X*KP*C13Y*W2*D43Y*P2Y3-2*X*KP*C13Y*W2
X*D43Y*P2Y5+2*D48*C11*XNUS*KP*W3*P2Y2-2*D48*C11*XNUS*KP*W3*P2Y0+2*
XD48*C11*XNUS*KP*W2*P2Y1-2*D48*C11*XNUS*KP*W2*P2Y3-2*D48*XNUS**3*
XKP**3*P8Y2*C14*W6
 FTOUT=FTOUT+2*D48*XNUS**3*KP**3*P8Y3*C14*W7+4*D48*XNUS**3*KP**3*
XC14*W7*P5Y3-4*D48*XNUS**3*KP**3*C14*W7*P2Y3-4*D48*XNUS**3*KP**3*
XC14*W6*P5Y2+4*D48*XNUS**3*KP**3*C14*W6*P2Y2+2*D48*XNUS**2*KP**2*
XC7*W4*P5Y1-2*D48*XNUS**2*KP**2*C7*W4*P5Y3-2*D48*XNUS**2*KP**2*C9*
XW4*P5Y2+2*D48*XNUS**2*KP**2*C9*W4*P5Y0+2*D48*XNUS**2*KP**2*C15*W5
X*P5Y3-2*D48*XNUS**2*KP**2*C15*W4*P5Y2+2*D48*XNUS**2*KP**2*C16*W5*
XP5Y2-2*D48*XNUS**2*KP**2*C16*W4*P5Y3-2*D48*XNUS**2*KP**2*C13*W5*
XP5Y1+2*D48*XNUS**2*KP**2*C13*W5*P5Y3-2*D48*XNUS**2*KP**2*C13*W4*
XP5Y2+2*D48*XNUS**2*KP**2*C13*W4*P5Y0-2*D48*XNUS**2*KP**2*C13Y*W5*
XP5Y2
 FTOUT=FTOUT+2*D48*XNUS**2*KP**2*C13Y*W5*P5Y4+2*D48*XNUS**2*KP**2*
XC13Y*W4*P5Y1-2*D48*XNUS**2*KP**2*C13Y*W4*P5Y3-2*D48*XNUS*KP**3*
XP8Y1*C14*W7-2*D48*XNUS*KP**3*C14*W7*P5Y1+4*D48*XNUS*KP**3*C14*W7*
XP2Y1+2*D48*XNUS*KP**3*C14*W6*P8Y0+2*D48*XNUS*KP**3*C14*W6*P5Y0-4*
XD48*XNUS*KP**3*C14*W6*P2Y0+1400*D48*XNUS*KP*P0*C14*W6-2*D48*XNUS*
XKP*C4*W2*P2Y2+2*D48*XNUS*KP*C4*W2*P2Y0-2*D48*XNUS*KP*C14*W7*P4Y1+
X2*D48*XNUS*KP*C14*W7*P3Y1+2*D48*XNUS*KP*C14*W6*P7Y2+2*D48*XNUS*KP
X*C14*W6*P4Y2-2*D48*XNUS*KP*C14*W6*P7Y0-2*D48*XNUS*KP*C14*W6*P3Y0-
X2*D48*XNUS*KP*C10Y*W3*P2Y2+2*D48*XNUS*KP*C10Y*W3*P2Y4+2*D48*XNUS*
XKP*C10Y*W2*P2Y1
 FTOUT=FTOUT-2*D48*XNUS*KP*C10Y*W2*P2Y3-2*D48*XNUS*KP*C11Y*W3*P2Y1+
X2*D48*XNUS*KP*C11Y*W3*P2Y3+2*D48*XNUS*KP*C11Y*W2*P2Y2-2*D48*XNUS*
XKP*C11Y*W2*P2Y4-2*D48*XNUS*KP*W3*C10*P2Y1+2*D48*XNUS*KP*W3*C10*
XP2Y3-2*D48*XNUS*KP*W2*C10*P2Y2+2*D48*XNUS*KP*W2*C10*P2Y0+2*D48*C9
X*W4*P4Y2-2*D48*C9*W4*P4Y0-2*D48*C15*W5*P4Y1+2*D48*C15*W5*P3Y1+2*
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XD48*C15*W4*P4Y0-2*D48*C15*W4*P3Y0-2*D48*C16*W5*P4Y0+2*D48*C16*W5*
XP3Y0+2*D48*C16*W4*P4Y1-2*D48*C16*W4*P3Y1+2*D48*C13*W4*P4Y2-2*D48*
XC13*W4*P4Y0-2*D48*C13Y*W4*P4Y1+2*D48*C13Y*W4*P4Y3-2*C11*P1Y1*D43-
X2*C11*P1Y1*D53Y+2*C11*P1Y2*D53-2*C11*P1Y2*D43Y+2*C11*P1Y3*D43+2*
XC11*P1Y3*D53Y
 FTOUT=FTOUT+2*C11*P1Y4*D43Y-2*C11*D53*P1Y0-2*P1Y1*C4*D43Y-2*P1Y1*
XC10Y*D43-2*P1Y1*C11Y*D53-2*P1Y1*D53*C10-2*P1Y1*D43Y*C10+2*P1Y2*C4
X*D43-2*P1Y2*C10Y*D53-2*P1Y2*C10Y*D43Y-2*P1Y2*C11Y*D43-2*P1Y2*C11Y
X*D53Y+2*P1Y2*D43*C10-2*P1Y2*D53Y*C10+2*P1Y3*C4*D43Y+2*P1Y3*C10Y*
XD43-2*P1Y3*C10Y*D53Y+2*P1Y3*C11Y*D53-2*P1Y3*C11Y*D43Y+2*P1Y3*D53*
XC10+2*P1Y3*D43Y*C10+2*XNUS**2*KP**2*P8Y1*C14*W4*D53+2*XNUS**2*KP
X**2*P8Y1*C14*W4*D43Y-2*XNUS**2*KP**2*P8Y2*C14*W4*D43+2*XNUS**2*KP
X**2*P8Y2*C14*W4*D53Y-2*XNUS**2*KP**2*P8Y3*C14*W4*D53-2*XNUS**2*KP
X**2*P8Y3*C14*W4*D43Y-2*XNUS**2*KP**2*C14*P8Y4*W4*D53Y+2*XNUS**2*
XKP**2*C14*W4*D43*P8Y0
 FTOUT=FTOUT+2*XNUS*KP*C7*W2*D43*P2Y1-2*XNUS*KP*C7*W2*D43*P2Y3+2*
XXNUS*KP*C7*W2*D43Y*P2Y2-2*XNUS*KP*C7*W2*D43Y*P2Y4-2*XNUS*KP*C9*W2
X*D43*P2Y2+2*XNUS*KP*C9*W2*D43*P2Y0+2*XNUS*KP*C9*W2*D43Y*P2Y1-2*
XXNUS*KP*C9*W2*D43Y*P2Y3+2*XNUS*KP*C15*D53*W2*P2Y1-2*XNUS*KP*C15*
XD53*W2*P2Y3-2*XNUS*KP*C15*W2*D43*P2Y2+2*XNUS*KP*C15*W2*D43*P2Y0+2
X*XNUS*KP*C15*W2*D53Y*P2Y2-2*XNUS*KP*C15*W2*D53Y*P2Y4+2*XNUS*KP*
XC15*W2*D43Y*P2Y1-2*XNUS*KP*C15*W2*D43Y*P2Y3-2*XNUS*KP*C16*D53*W2*
XP2Y2+2*XNUS*KP*C16*D53*W2*P2Y0+2*XNUS*KP*C16*W2*D43*P2Y1-2*XNUS*
XKP*C16*W2*D43*P2Y3+2*XNUS*KP*C16*W2*D53Y*P2Y1-2*XNUS*KP*C16*W2*
XD53Y*P2Y3
 FTOUT=FTOUT+2*XNUS*KP*C16*W2*D43Y*P2Y2-2*XNUS*KP*C16*W2*D43Y*P2Y4+
X2*XNUS*KP*C13*D53*W2*P2Y1-2*XNUS*KP*C13*D53*W2*P2Y3-2*XNUS*KP*C13
X*W2*D43*P2Y2+2*XNUS*KP*C13*W2*D43*P2Y0+2*XNUS*KP*C13*W2*D53Y*P2Y2
X-2*XNUS*KP*C13*W2*D53Y*P2Y4+2*XNUS*KP*C13*W2*D43Y*P2Y1-2*XNUS*KP*
XC13*W2*D43Y*P2Y3+2*XNUS*KP*C13Y*D53*W2*P2Y2-2*XNUS*KP*C13Y*D53*W2
X*P2Y4+2*XNUS*KP*C13Y*W2*D43*P2Y1-2*XNUS*KP*C13Y*W2*D43*P2Y3+2*
XXNUS*KP*C13Y*W2*D53Y*P2Y3-2*XNUS*KP*C13Y*W2*D53Y*P2Y5+2*XNUS*KP*
XC13Y*W2*D43Y*P2Y2-2*XNUS*KP*C13Y*W2*D43Y*P2Y4+2*P1Y4*C10Y*D53+2*
XP1Y4*C10Y*D43Y+2*P1Y4*C11Y*D43+2*P1Y4*C11Y*D53Y+2*P1Y4*D53Y*C10+2
X*P1Y5*C10Y*D53Y
 FTOUT=FTOUT+2*P1Y5*C11Y*D43Y-2*C4*D43*P1Y0-2*C14*W4*P7Y1*D43Y+2*
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XC14*W4*D43*P7Y2-2*C14*W4*D43*P7Y0+2*C14*W4*P7Y3*D43Y-2*D43*C10*
XP1Y0
 FTOUT=
XFTOUT-2*LAW*C20*D10*P1Y0+2*LAW*C20*D10*P1Y2-2*L*W*C20*D10Y*
XP1Y1+2*L*W*C20*D10Y*P1Y3-2*LAW*D10*C20Y*P1Y1+2*L*W*D10*C20Y*
XP1Y3-2*L*W*C20Y*D10Y*P1Y2+2*L*W*C20Y*D10Y*P1Y4-2*L*W*C27*D11*
XP1Y0+2*L*W*C27*D11*P1Y2-2*L*W*C27*D11Y*P1Y1+2*L*W*C27*D11Y*P1Y3
 FTOUT=FTOUT
X+2*L*X**2*P5Y1*W4*D5*C37*KP**2-2*L*X**2*P5Y0*D5*C37*KP**2*W5-4*
XL*X*P5Y1*D5*C37*XNUS*KP**2*W5+2*L*X*W4*P5Y2*D5*C37*XNUS*KP**2+2
X*L*X*W4*P5Y0*D5*C37*XNUS*KP**2+2*L*X*W3*D10*C22*P2Y0*KP-2*L*X*
XW3*D10*C22*P2Y2*KP+2*L*X*W3*D10*C24*P2Y1*KP-2*L*X*W3*D10*C24*
XP2Y3*KP+2*L*X*W3*D10*C28*P2Y0*KP-2*L*X*W3*D10*C28*P2Y2*KP+2*L*X
X*W3*C22*D10Y*P2Y1*KP-2*L*X*W3*C22*D10Y*P2Y3*KP+2*L*X*W3*C24*
XD10Y*P2Y2*KP-2*L*X*W3*C24*D10Y*P2Y4*KP+2*L*X*W3*D10Y*C28*P2Y1*
XKP-2*L*X*W3*D10Y*C28*P2Y3*KP-2*L*X*W2*D10*C28*P2Y1*KP+2*L*X*W2*
XD10*C28*P2Y3*KP+2*L*X*W2*D11*C22*P2Y0*KP-2*L*X*W2*D11*C22*P2Y2*
XKP+2*L*X*W2*D11*C24*P2Y1*KP-2*L*X*W2*D11*C24*P2Y3*KP+2*L*X*W2*
XC22*D11Y*P2Y1*KP-2*L*X*W2*C22*D11Y*P2Y3*KP+2*L*X*W2*C24*D11Y*
XP2Y2*KP-2*L*X*W2*C24*D11Y*P2Y4*KP-2*L*X*W2*D10Y*C28*P2Y2*KP+2*L
X*X*W2*D10Y*C28*P2Y4*KP+2*L*P5Y1*W4*D5*C37*XNUS**2*KP**2-2*L*W4*
XP4Y1*D5*C37+2*L*W4*P3Y1*D5*C37-2*L*P5Y2*D5*C37*XNUS**2*KP**2*W5
X+2*L*W3*D10*C22*P2Y1*XNUS*KP-2*L*W3*D10*C22*P2Y3*XNUS*KP+2*L*W3
X*D10*C24*P2Y0*XNUS*KP
  FTOUT=FTOUT-2*L*W3*D10*C24*P2Y2*XNUS*KP-2*L*W3*D11*C28*P2Y0*XNUS
X*KP+2*L*W3*D11*C28*P2Y2*XNUS*KP+2*L*W3*C22*D10Y*P2Y2*XNUS*KP-2*
XL*W3*C22*D10Y*P2Y4*XNUS*KP+2*L*W3*C24*D10Y*P2Y1*XNUS*KP-2*L*W3*
XC24*D10Y*P2Y3*XNUS*KP-2*L*W3*C28*D11Y*P2Y1*XNUS*KP+2*L*W3*C28*
XD11Y*P2Y3*XNUS*KP+2*L*W3*P2Y0*D5*XNUS*KP*C21-2*L*W3*D5*P2Y2*
XXNUS*KP*C21+2*L*W2*D11*C22*P2Y1*XNUS*KP-2*L*W2*D11*C22*P2Y3*
XXNUS*KP+2*L*W2*D11*C24*P2Y0*XNUS*KP-2*L*W2*D11*C24*P2Y2*XNUS*KP
X+2*L*W2*D11*C28*P2Y1*XNUS*KP-2*L*W2*D11*C28*P2Y3*XNUS*KP+2*L*W2
X*C22*D11Y*P2Y2*XNUS*KP-2*L*W2*C22*D11Y*P2Y4*XNUS*KP+2*L*W2*C24*
XD11Y*P2Y1*XNUS*KP-2*L*W2*C24*D11Y*P2Y3*XNUS*KP+2*L*W2*C28*D11Y*
XP2Y2*XNUS*KP-2*L*W2*C28*D11Y*P2Y4*XNUS*KP-2*L*C20*D11*P1Y0+2*L*
XC20*D11*P1Y2-2*L*C20*D11Y*P1Y1+2*L*C20*D11Y*P1Y3-2*L*C20Y*D11*
```

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XP1Y1+2*L*C20Y*D11*P1Y3-2*L*C20Y*D11Y*P1Y2+2*L*C20Y*D11Y*P1Y4+2*
XL*C27*D11*P1Y1-2*L*C27*D11*P1Y3+2*L*C27*D11Y*P1Y2-2*L*C27*D11Y*
XP1Y4+2*L*P4Y0*D5*C37*W5-2*L*P3Y0*D5*C37*W5+2*W*D10*C41*P1Y0-2*W
X*D10*C41*P1Y2+2*W*D10*C42*P1Y1-2*W*D10*C42*P1Y3+2*W*C41*D10Y*
XP1Y1-2*W*C41*D10Y*P1Y3+2*W*C42*D11*P1Y0-2*W*C42*D11*P1Y2+2*W*
XC42*D10Y*P1Y2-2*W*C42*D10Y*P1Y4+2*W*C42*D11Y*P1Y1-2*W*C42*D11Y*
XP1Y3-4*X**3*P5Y1*D5*C52*KP**3*W6+2*X**3*P8Y0*D5*C52*KP**3*W7+4*
XX**3*P5Y0*D5*C52*KP**3*W7
 FTOUT=FTOUT-4*X**3*P2Y0*D5*C52*KP**3*W7+4*X**3*D5*C52*P2Y1*KP**3
X*W6-2*X**3*D5*C52*P8Y1*KP**3*W6-2*X**2*P5Y1*W4*D5*C50*KP**2+8*X
X**2*P5Y1*D5*C52*XNUS*KP**3*W7-4*X**2*P5Y1*D5*C52*XNUS*KP**3*W6-
X4*X**2*P5Y2*D5*C52*XNUS*KP**3*W6+4*X**2*P8Y0*D5*C52*XNUS*KP**3*
XW7-2*X**2*P8Y0*D5*C52*XNUS*KP**3*W6+4*X**2*P5Y0*D5*C52*XNUS*KP
X**3*W7-4*X**2*P5Y0*D5*C52*XNUS*KP**3*W6+2*X**2*P5Y0*D5*C50*KP**
X2*W5-8*X**2*P2Y0*D5*C52*XNUS*KP**3*W7+4*X**2*P2Y0*D5*C52*XNUS*
XKP**3*W6-4*X**2*D5*C52*P2Y1*XNUS*KP**3*W7+8*X**2*D5*C52*P2Y1*
XXNUS*KP**3*W6+2*X**2*D5*C52*P8Y1*XNUS*KP**3*W7-4*X**2*D5*C52*
XP8Y1*XNUS*KP**3*W6+2*X*P5Y1*W4*C47*D5*XNUS*KP**2+4*X*P5Y1*D5*
XC52*XNUS**2*KP**3*W7-4*X*P5Y1*D5*C52*XNUS**2*KP**3*W6+2*X*P5Y1*
XD5*C52*KP**3*W6+4*X*P5Y1*D5*C50*XNUS*KP**2*W5-2*X*W4*P5Y2*C45*
XD5*XNUS*KP**2-4*X*W4*P5Y2*D5*C50*XNUS*KP**2-2*X*W4*P5Y3*C47*D5*
XXNUS*KP**2+2*X*W4*C45*P5Y0*D5*XNUS*KP**2+8*X*P5Y2*D5*C52*XNUS**
X2*KP**3*W7-2*X*W3*D10*C44*P2Y0*KP+2*X*W3*D10*C44*P2Y2*KP-2*X*W3
X*C44*D10Y*P2Y1*KP+2*X*W3*C44*D10Y*P2Y3*KP-4*X*P5Y3*D5*C52*XNUS
X**2*KP**3*W6+2*X*W2*D10*C40*P2Y1*KP-2*X*W2*D10*C40*P2Y3*KP+2*X*
XW2*D10*C44*P2Y1*KP-2*X*W2*D10*C44*P2Y3*KP+2*X*W2*D10*C40Y*P2Y2*
XKP-2*X*W2*D10*C40Y*P2Y4*KP+2*X*W2*C40*D11*P2Y0*KP-2*X*W2*C40*
XD11*P2Y2*KP
 FTOUT=FTOUT+2*X*W2*C40*D10Y*P2Y2*KP-2*X*W2*C40*D10Y*P2Y4*KP+2*X*
XW2*C40*D11Y*P2Y1*KP-2*X*W2*C40*D11Y*P2Y3*KP+2*X*W2*D11*C40Y*
XP2Y1*KP-2*X*W2*D11*C40Y*P2Y3*KP+2*X*W2*C44*D10Y*P2Y2*KP-2*X*W2*
XC44*D10Y*P2Y4*KP+2*X*W2*D10Y*C40Y*P2Y3*KP-2*X*W2*D10Y*C40Y*P2Y5
X*KP+2*X*W2*C40Y*D11Y*P2Y2*KP-2*X*W2*C40Y*D11Y*P2Y4*KP+2*X*P4Y1*
XD5*C52*KP*W6-2*X*P3Y1*D5*C52*KP*W6-4*X*P8Y0*D5*C52*XNUS**2*KP**
X3*W6-2*X*P8Y0*D5*C52*KP**3*W7-4*X*P5Y0*D5*C52*XNUS**2*KP**3*W6-
X2*X*P5Y0*D5*C52*KP**3*W7-2*X*P4Y0*D5*C52*KP*W7+2*X*P3Y0*D5*C52*
```

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XKP*W7+8*X*P2Y0*D5*C52*XNUS**2*KP**3*W6+4*X*P2Y0*D5*C52*KP**3*W7
X-8*X*D5*C52*P2Y1*XNUS**2*KP**3*W7-4*X*D5*C52*P2Y1*KP**3*W6-4*X*
XD5*C52*P2Y2*XNUS**2*KP**3*W7+4*X*D5*C52*P2Y3*XNUS**2*KP**3*W6+4
X*X*D5*C52*P8Y1*XNUS**2*KP**3*W7+2*X*D5*C52*P8Y1*KP**3*W6+2*X*D5
X*C52*P8Y2*XNUS**2*KP**3*W7-2*X*D5*C52*P8Y3*XNUS**2*KP**3*W6-840
X*X*D5*C52*KP*P0*W7+2*P5Y1*W4*C45*D5*XNUS**2*KP**2-2*P5Y1*D5*C52
X*XNUS*KP**3*W7-2*W4*P5Y2*C47*D5*XNUS**2*KP**2-2*W4*P5Y3*C45*D5*
XXNUS**2*KP**2-2*W4*P5Y3*D5*C50*XNUS**2*KP**2+2*W4*P4Y1*D5*C50+2
X*W4*P4Y2*C47*D5-2*W4*P3Y1*D5*C50+2*W4*C47*P5Y0*D5*XNUS**2*KP**2
X-2*W4*C47*P4Y0*D5-4*P5Y2*D5*C52*XNUS**3*KP**3*W6+2*P5Y2*D5*C50*
XXNUS**2*KP**2*W5-2*W3*D10*C48*P2Y1*XNUS*KP+2*W3*D10*C48*P2Y3*
XXNUS*KP
  FTOUT=FTOUT-2*W3*D10*C44*P2Y1*XNUS*KP+2*W3*D10*C44*P2Y3*XNUS*KP-
X2*W3*C48*D11*P2Y0*XNUS*KP+2*W3*C48*D11*P2Y2*XNUS*KP-2*W3*C48*
XD10Y*P2Y2*XNUS*KP+2*W3*C48*D10Y*P2Y4*XNUS*KP-2*W3*C48*D11Y*P2Y1
X*XNUS*KP+2*W3*C48*D11Y*P2Y3*XNUS*KP-2*W3*C44*D10Y*P2Y2*XNUS*KP+
X2*W3*C44*D10Y*P2Y4*XNUS*KP+4*P5Y3*D5*C52*XNUS**3*KP**3*W7+2*W2*
XC39*P2Y0*D5*XNUS*KP-2*W2*C39*D5*P2Y2*XNUS*KP+2*W2*D10*C48*P2Y0*
XXNUS*KP-2*W2*D10*C48*P2Y2*XNUS*KP+2*W2*D10*C40*P2Y0*XNUS*KP-2*
XW2*D10*C40*P2Y2*XNUS*KP+2*W2*D10*C44*P2Y0*XNUS*KP-2*W2*D10*C44*
XP2Y2*XNUS*KP+2*W2*D10*C40Y*P2Y1*XNUS*KP-2*W2*D10*C40Y*P2Y3*XNUS
X*KP+2*W2*C48*D11*P2Y1*XNUS*KP-2*W2*C48*D11*P2Y3*XNUS*KP+2*W2*
XC48*D10Y*P2Y1*XNUS*KP-2*W2*C48*D10Y*P2Y3*XNUS*KP+2*W2*C48*D11Y*
XP2Y2*XNUS*KP-2*W2*C48*D11Y*P2Y4*XNUS*KP+2*W2*C40*D11*P2Y1*XNUS*
XKP-2*W2*C40*D11*P2Y3*XNUS*KP+2*W2*C40*D10Y*P2Y1*XNUS*KP-2*W2*
XC40*D10Y*P2Y3*XNUS*KP+2*W2*C40*D11Y*P2Y2*XNUS*KP-2*W2*C40*D11Y*
XP2Y4*XNUS*KP+2*W2*D11*C40Y*P2Y2*XNUS*KP-2*W2*D11*C40Y*P2Y4*XNUS
X*KP+2*W2*C44*D10Y*P2Y1*XNUS*KP-2*W2*C44*D10Y*P2Y3*XNUS*KP+2*W2*
XD10Y*C40Y*P2Y2*XNUS*KP-2*W2*D10Y*C40Y*P2Y4*XNUS*KP+2*W2*C40Y*
XD11Y*P2Y3*XNUS*KP-2*W2*C40Y*D11Y*P2Y5*XNUS*KP-2*C53*D10*P1Y1+2*
XC53*D10*P1Y3-2*C53*D11*P1Y0+2*C53*D11*P1Y2-2*C53*D10Y*P1Y2+2*
XC53*D10Y*P1Y4
  FTOUT=FTOUT-2*C53*D11Y*P1Y1+2*C53*D11Y*P1Y3-2*D10*C43*P1Y0+2*D10
X*C43*P1Y2-2*D10*C41*P1Y1+2*D10*C41*P1Y3-2*D10*C42*P1Y0+2*D10*
XC42*P1Y2-2*D10*C53Y*P1Y2+2*D10*C53Y*P1Y4-2*D10*C43Y*P1Y1+2*D10*
XC43Y*P1Y3-2*C43*D11*P1Y1+2*C43*D11*P1Y3-2*C43*D10Y*P1Y1+2*C43*
```

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XD10Y*P1Y3-2*C43*D11Y*P1Y2+2*C43*D11Y*P1Y4-2*P4Y1*D5*C52*XNUS*KP
X*W7+2*P4Y2*D5*C52*XNUS*KP*W6-2*C41*D10Y*P1Y2+2*C41*D10Y*P1Y4-2*
XC42*D11*P1Y1+2*C42*D11*P1Y3-2*C42*D10Y*P1Y1+2*C42*D10Y*P1Y3-2*
XC42*D11Y*P1Y2+2*C42*D11Y*P1Y4-2*C53Y*D11*P1Y1+2*C53Y*D11*P1Y3-2
X*C53Y*D10Y*P1Y3+2*C53Y*D10Y*P1Y5-2*C53Y*D11Y*P1Y2+2*C53Y*D11Y*
XP1Y4-2*D11*C43Y*P1Y2+2*D11*C43Y*P1Y4-2*C43Y*D10Y*P1Y2+2*C43Y*
XD10Y*P1Y4-2*C43Y*D11Y*P1Y3+2*C43Y*D11Y*P1Y5+2*P3Y1*D5*C52*XNUS*
XKP*W7+2*P8Y0*D5*C52*XNUS*KP**3*W6-2*P7Y0*D5*C52*XNUS*KP*W6+2*
XP5Y0*D5*C52*XNUS*KP**3*W6-2*P4Y0*D5*C50*W5-2*P3Y0*D5*C52*XNUS*
XKP*W6+2*P3Y0*D5*C50*W5-4*P2Y0*D5*C52*XNUS*KP**3*W6+4*D5*C52*
XP2Y1*XNUS*KP**3*W7+4*D5*C52*P2Y2*XNUS**3*KP**3*W6-4*D5*C52*P2Y3
X*XNUS**3*KP**3*W7-2*D5*C52*P8Y1*XNUS*KP**3*W7-2*D5*C52*P8Y2*
XXNUS**3*KP**3*W6+2*D5*C52*P8Y3*XNUS**3*KP**3*W7+2*D5*C52*XNUS*
XP7Y2*KP*W6+1400*D5*C52*XNUS*KP*P0*W6
C                         TERM3
 FTOUT=
XFTOUT-2*L*W*D43Y*C11Y*P1Y3+2*L*W*D43Y*C11Y*P1Y5-2*L*W*D43Y*P1Y1*
XC4-2*L*W*D43Y*P1Y1*C10-2*L*W*D43Y*P1Y2*C11-2*L*W*D43Y*P1Y2*C10Y+2
X*L*W*D43Y*P1Y3*C4+2*L*W*D43Y*P1Y3*C10+2*L*W*D43Y*P1Y4*C11+2*L*W*
XD43Y*P1Y4*C10Y-2*L*W*C11Y*P1Y1*D53-2*L*W*C11Y*P1Y2*D43-2*L*W*C11Y
X*P1Y2*D53Y+2*L*W*C11Y*P1Y3*D53+2*L*W*C11Y*P1Y4*D43+2*L*W*C11Y*
XP1Y4*D53Y-2*L*W*P1Y1*C10*D53-2*L*W*P1Y1*C11*D43-2*L*W*P1Y1*C11*
XD53Y-2*L*W*P1Y1*D43*C10Y+2*L*W*P1Y2*C4*D43+2*L*W*P1Y2*C10*D43-2*L
X*W*P1Y2*C10*D53Y+2*L*W*P1Y2*D53*C11-2*L*W*P1Y2*D53*C10Y+2*L*W*
XP1Y3*C10*D53+2*L*W*P1Y3*C11*D43+2*L*W*P1Y3*C11*D53Y+2*L*W*P1Y3*
XD43*C10Y
  FTOUT=FTOUT-2*L*W*P1Y3*D53Y*C10Y+2*L*W*P1Y4*C10*D53Y+2*L*W*P1Y4*
XD53*C10Y+2*L*W*P1Y5*D53Y*C10Y-2*L*W*C4*D43*P1Y0-2*L*W*C10*D43*
XP1Y0-2*L*W*D53*C11*P1Y0+2*W*C1*P1Y1*D53Y-2*W*C1*P1Y2*D53-2*W*C1*
XP1Y3*D53Y+2*W*C1*D53*P1Y0+2*X*D43Y*P2Y1*C2*KP*W3-2*X*D43Y*C2*P2Y3
X*KP*W3-2*X*D48*C2*XNUS*KP**2*W5*P5Y0+2*X*D48*C2*XNUS*KP**2*W5*
XP5Y2-2*X*C2*P2Y2*KP*W3*D43+2*X*C2*KP*W3*D43*P2Y0-2*C1*D48*P2Y1*
XXNUS*KP*W3+2*C1*D48*P2Y3*XNUS*KP*W3-2*D48*C2*XNUS**2*KP**2*W5*
XP5Y1+2*D48*C2*XNUS**2*KP**2*W5*P5Y3-2*P2Y1*C2*XNUS*KP*W3*D53Y+2*
XC2*P2Y2*XNUS*KP*D53*W3+2*C2*P2Y3*XNUS*KP*W3*D53Y-2*C2*XNUS*KP*D53
X*W3*P2Y0
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 FTOUT=
XFTOUT-2*L*W*C53Y*D11Y*P1Y2-2*L*W*C53Y*D10*P1Y2-2*L*W*C53Y*D11*
XP1Y1-2*L*W*D11Y*C53*P1Y1-2*L*W*D11Y*C43*P1Y2-2*L*W*D10*C43Y*P1Y1-
X2*L*W*D10*C53*P1Y1+2*L*W*D10*C43*P1Y2-2*L*W*D10*C43*P1Y0-2*L*W*
XD10Y*C43Y*P1Y2-2*L*W*D10Y*C53*P1Y2-2*L*W*D10Y*C43*P1Y1-2*L*W*C43Y
X*D11*P1Y2+2*L*W*C53*D11*P1Y2-2*L*W*C53*D11*P1Y0-2*L*W*D11*C43*
XP1Y1-2*W*D11Y*C20*P1Y1+2*W*D11Y*C20*P1Y3-2*W*D11Y*C20Y*P1Y2+2*W*
XD11Y*C20Y*P1Y4-2*W*D11Y*C30*P1Y1+4*W*D11Y*C30*P1Y3-2*W*D11Y*C30*
XP1Y5-2*W*C20*D10*P1Y1+2*W*C20*D10*P1Y3-2*W*C20*D10Y*P1Y2+2*W*C20*
XD10Y*P1Y4+2*W*C20*D11*P1Y2-2*W*C20*D11*P1Y0-2*W*C20Y*D10*P1Y2+2*W
X*C20Y*D10*P1Y4
 FTOUT=FTOUT-2*W*C20Y*D10Y*P1Y3+2*W*C20Y*D10Y*P1Y5-2*W*C20Y*D11*
XP1Y1+2*W*C20Y*D11*P1Y3-4*W*D10*C31*P1Y2+2*W*D10*C31*P1Y4+2*W*D10*
XC31*P1Y0+4*W*C30*D11*P1Y2-2*W*C30*D11*P1Y4-2*W*C30*D11*P1Y0+2*W*
XD10Y*C31*P1Y1-4*W*D10Y*C31*P1Y3+2*W*D10Y*C31*P1Y5+2*X*D11Y*C22*
XP2Y1*KP*W3-2*X*D11Y*C22*P2Y3*KP*W3+2*X*D11Y*C24*P2Y2*KP*W3-2*X*
XD11Y*C24*P2Y4*KP*W3+2*X*C22*D10*P2Y1*KP*W3-2*X*C22*D10*P2Y3*KP*W3
X+2*X*C22*D10Y*P2Y2*KP*W3-2*X*C22*D10Y*P2Y4*KP*W3-2*X*C22*D11*P2Y2
X*KP*W3+2*X*C22*D11*KP*P2Y0*W3+2*X*C24*D10*P2Y2*KP*W3-2*X*C24*D10*
XP2Y4*KP*W3+2*X*C24*D10Y*P2Y3*KP*W3-2*X*C24*D10Y*P2Y5*KP*W3+2*X*
XC24*D11*P2Y1*KP*W3
  FTOUT=FTOUT-2*X*C24*D11*P2Y3*KP*W3-2*X*P5Y2*C37*D5*XNUS*KP**2*W5+4
X*X*C29*D10*P2Y2*KP*W3-2*X*C29*D10*P2Y4*KP*W3-2*X*C29*D10*KP*P2Y0*
XW3-2*X*C29*D10Y*P2Y1*KP*W3+4*X*C29*D10Y*P2Y3*KP*W3-2*X*C29*D10Y*
XP2Y5*KP*W3+2*X*C37*D5*XNUS*KP**2*P5Y0*W5+2*D11Y*C22*P2Y2*XNUS*KP*
XW3-2*D11Y*C22*P2Y4*XNUS*KP*W3+2*D11Y*C24*P2Y1*XNUS*KP*W3-2*D11Y*
XC24*P2Y3*XNUS*KP*W3+2*D11Y*C38*P2Y1*XNUS*KP*W3-4*D11Y*C38*P2Y3*
XXNUS*KP*W3+2*D11Y*C38*P2Y5*XNUS*KP*W3+2*D11Y*C29*P2Y1*XNUS*KP*W3-
X4*D11Y*C29*P2Y3*XNUS*KP*W3+2*D11Y*C29*P2Y5*XNUS*KP*W3+2*C21*D5*
XP2Y1*XNUS*KP*W3-2*C21*D5*P2Y3*XNUS*KP*W3+2*C22*D10*P2Y2*XNUS*KP*
XW3
  FTOUT=FTOUT-2*C22*D10*P2Y4*XNUS*KP*W3+2*C22*D10Y*P2Y3*XNUS*KP*W3-2
X*C22*D10Y*P2Y5*XNUS*KP*W3+2*C22*D11*P2Y1*XNUS*KP*W3-2*C22*D11*
XP2Y3*XNUS*KP*W3+2*P5Y1*C37*D5*XNUS**2*KP**2*W5+2*C24*D10*P2Y1*
XXNUS*KP*W3-2*C24*D10*P2Y3*XNUS*KP*W3+2*C24*D10Y*P2Y2*XNUS*KP*W3-2
X*C24*D10Y*P2Y4*XNUS*KP*W3-2*C24*D11*P2Y2*XNUS*KP*W3+2*C24*D11*
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XXNUS*KP*P2Y0*W3-2*P5Y3*C37*D5*XNUS**2*KP**2*W5-4*C38*D11*P2Y2*
XXNUS*KP*W3+2*C38*D11*P2Y4*XNUS*KP*W3+2*C38*D11*XNUS*KP*P2Y0*W3-4*
XC29*D11*P2Y2*XNUS*KP*W3+2*C29*D11*P2Y4*XNUS*KP*W3+2*C29*D11*XNUS*
XKP*P2Y0*W3
 FTOUT=
XFTOUT+2*L*W*D10*C53*P1Y3+2*L*W*D10*C53Y*P1Y4+2*L*W*D10*C43Y*P1Y3+
X2*L*W*D11*C43*P1Y3+2*L*W*D11*C53Y*P1Y3+2*L*W*D11*C43Y*P1Y4+2*L*W*
XD10Y*C53*P1Y4+2*L*W*D10Y*C43*P1Y3-2*L*W*D10Y*C53Y*P1Y3+2*L*W*D10Y
X*C53Y*P1Y5+2*L*W*D10Y*C43Y*P1Y4+2*L*W*D11Y*C53*P1Y3+2*L*W*D11Y*
XC43*P1Y4+2*L*W*D11Y*C53Y*P1Y4-2*L*W*D11Y*C43Y*P1Y3+2*L*W*D11Y*
XC43Y*P1Y5+4*L*X*C52*D5*P5Y1*W7*XNUS**2*KP**3+4*L*X*C52*D5*P5Y2*W7
X*XNUS**2*KP**3-4*L*X*C52*D5*P5Y3*W7*XNUS**2*KP**3+4*L*X*C52*D5*W7
X*XNUS**2*KP**3*P8Y0-4*L*X*C52*D5*W7*XNUS**2*KP**3*P5Y0-4*L*X*C52*
XD5*W7*XNUS**2*KP**3*P8Y1-4*L*X*C52*D5*W7*XNUS**2*KP**3*P8Y2+4*L*X
X*C52*D5*W7*XNUS**2*KP**3*P8Y3
  FTOUT=FTOUT+2*L*X*D5*P5Y1*C47*W5*XNUS*KP**2-2*L*X*D5*P5Y2*C45*W5*
 XXNUS*KP**2-2*L*X*D5*P5Y3*C47*W5*XNUS*KP**2+2*L*X*D5*C45*W5*XNUS*
 XKP**2*P5Y0+2*L*X*D10*C40*P2Y1*W3*KP-2*L*X*D10*C40*P2Y3*W3*KP+2*L*
 XX*D10*C40Y*P2Y2*W3*KP-2*L*X*D10*C40Y*P2Y4*W3*KP-2*L*X*D11*C40*
 XP2Y2*W3*KP+2*L*X*D11*C40*W3*KP*P2Y0+2*L*X*D11*C40Y*P2Y1*W3*KP-2*L
 X*X*D11*C40Y*P2Y3*W3*KP+2*L*X*C40*D10Y*P2Y2*W3*KP-2*L*X*C40*D10Y*
 XP2Y4*W3*KP+2*L*X*C40*D11Y*P2Y1*W3*KP-2*L*X*C40*D11Y*P2Y3*W3*KP+2*
 XL*X*D10Y*C40Y*P2Y3*W3*KP-2*L*X*D10Y*C40Y*P2Y5*W3*KP+2*L*X*C40Y*
 XD11Y*P2Y2*W3*KP-2*L*X*C40Y*D11Y*P2Y4*W3*KP-2*L*C52*D5*P5Y2*W7*
 XXNUS*KP**3
  FTOUT=FTOUT+4*L*C52*D5*P4Y2*W7*XNUS*KP-2*L*C52*D5*P3Y2*W7*XNUS*KP-
 X2*L*C52*D5*W7*XNUS*KP**3*P8Y0+2*L*C52*D5*W7*XNUS*KP**3*P5Y0+2*L*
 XC52*D5*W7*XNUS*KP**3*P8Y2+4*L*C52*D5*W7*XNUS*KP*P7Y0-2*L*C52*D5*
 XW7*XNUS*KP*P6Y0-4*L*C52*D5*W7*XNUS*KP*P4Y0+2*L*C52*D5*W7*XNUS*KP*
 XP3Y0-4*L*C52*D5*W7*XNUS*KP*P7Y2+2*L*C52*D5*W7*XNUS*KP*P6Y2+2*L*D5
 X*P5Y1*C45*W5*XNUS**2*KP**2-2*L*D5*P5Y2*C47*W5*XNUS**2*KP**2-2*L*
 XD5*P5Y3*C45*W5*XNUS**2*KP**2-2*L*D5*C39*P2Y2*W3*XNUS*KP+2*L*D5*
 XC39*W3*XNUS*KP*P2Y0+2*L*D5*P4Y2*C47*W5+2*L*D5*C47*W5*XNUS**2*KP**
 X2*P5Y0-2*L*D5*C47*W5*P4Y0-2*L*D10*C40*P2Y2*W3*XNUS*KP+2*L*D10*C40
 X*W3*XNUS*KP*P2Y0
  FTOUT=FTOUT+2*L*D10*C40Y*P2Y1*W3*XNUS*KP-2*L*D10*C40Y*P2Y3*W3*XNUS
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X*KP+2*L*D11*C40*P2Y1*W3*XNUS*KP-2*L*D11*C40*P2Y3*W3*XNUS*KP+2*L*
XD11*C40Y*P2Y2*W3*XNUS*KP-2*L*D11*C40Y*P2Y4*W3*XNUS*KP+2*L*C40*
XD10Y*P2Y1*W3*XNUS*KP-2*L*C40*D10Y*P2Y3*W3*XNUS*KP+2*L*C40*D11Y*
XP2Y2*W3*XNUS*KP-2*L*C40*D11Y*P2Y4*W3*XNUS*KP+2*L*D10Y*C40Y*P2Y2*
XW3*XNUS*KP-2*L*D10Y*C40Y*P2Y4*W3*XNUS*KP+2*L*C40Y*D11Y*P2Y3*W3*
XXNUS*KP-2*L*C40Y*D11Y*P2Y5*W3*XNUS*KP
 FTOUT=
XFTOUT-2*L*X**2*D48*C13*P5Y0*W5*KP**2+2*L*X**2*D48*C13*W5*KP**2*
XP5Y2-2*L*X**2*D48*C13Y*W5*KP**2*P5Y1+2*L*X**2*D48*C13Y*W5*KP**2*
XP5Y3-4*L*X*D48*C14*P8Y0*W7*XNUS**2*KP**3+4*L*X*D48*C14*P5Y0*W7*
XXNUS**2*KP**3+4*L*X*D48*C14*W7*P8Y1*XNUS**2*KP**3+4*L*X*D48*C14*
XW7*P8Y2*XNUS**2*KP**3-4*L*X*D48*C14*W7*P8Y3*XNUS**2*KP**3-4*L*X*
XD48*C14*W7*XNUS**2*KP**3*P5Y1-4*L*X*D48*C14*W7*XNUS**2*KP**3*P5Y2
X+4*L*X*D48*C14*W7*XNUS**2*KP**3*P5Y3-2*L*X*D48*C7*P5Y0*W5*XNUS*KP
X**2+2*L*X*D48*C7*W5*XNUS*KP**2*P5Y2-2*L*X*D48*C9*W5*XNUS*KP**2*
XP5Y1+2*L*X*D48*C9*W5*XNUS*KP**2*P5Y3-4*L*X*D48*C13*W5*XNUS*KP**2*
XP5Y1
  FTOUT=FTOUT+4*L*X*D48*C13*W5*XNUS*KP**2*P5Y3-4*L*X*D48*C13Y*W5*
XXNUS*KP**2*P5Y2+4*L*X*D48*C13Y*W5*XNUS*KP**2*P5Y4+2*L*X*C14*D43*
XW5*P8Y1*XNUS*KP**2-2*L*X*C14*D43*W5*P8Y3*XNUS*KP**2+2*L*X*C14*D53
X*P8Y0*W5*XNUS*KP**2-2*L*X*C14*D53*W5*P8Y2*XNUS*KP**2+2*L*X*C14*
XD43Y*W5*P8Y2*XNUS*KP**2-2*L*X*C14*D43Y*W5*P8Y4*XNUS*KP**2+2*L*X*
XC14*D53Y*W5*P8Y1*XNUS*KP**2-2*L*X*C14*D53Y*W5*P8Y3*XNUS*KP**2+2*L
X*X*P2Y1*C7*D43Y*W3*KP+2*L*X*P2Y1*C9*D43*W3*KP+2*L*X*P2Y1*D43*C13*
XW3*KP+2*L*X*P2Y1*D53*C13Y*W3*KP+2*L*X*P2Y1*D53Y*C13*W3*KP-2*L*X*
XP2Y2*C7*D43*W3*KP+2*L*X*P2Y2*C9*D43Y*W3*KP+2*L*X*P2Y2*D43*C13Y*W3
X*KP
  FTOUT=FTOUT-2*L*X*P2Y2*D53*C13*W3*KP+2*L*X*P2Y2*D43Y*C13*W3*KP+2*L
X*X*P2Y2*D53Y*C13Y*W3*KP-2*L*X*P2Y3*C7*D43Y*W3*KP-2*L*X*P2Y3*C9*
XD43*W3*KP-2*L*X*P2Y3*D43*C13*W3*KP-2*L*X*P2Y3*D53*C13Y*W3*KP+2*L*
XX*P2Y3*D43Y*C13Y*W3*KP-2*L*X*P2Y3*D53Y*C13*W3*KP-2*L*X*P2Y4*C9*
XD43Y*W3*KP-2*L*X*P2Y4*D43*C13Y*W3*KP-2*L*X*P2Y4*D43Y*C13*W3*KP-2*
XL*X*P2Y4*D53Y*C13Y*W3*KP+2*L*X*C7*D43*P2Y0*W3*KP-2*L*X*P2Y5*D43Y*
XC13Y*W3*KP+2*L*X*D53*C13*P2Y0*W3*KP+4*L*P4Y1*D48*C13Y*W5-4*L*P4Y2
X*D48*C14*W7*XNUS*KP-2*L*P4Y2*D48*C9*W5-4*L*P4Y2*D48*C13*W5+2*L*
XD48*C14*P3Y2*W7*XNUS*KP+2*L*D48*C14*P8Y0*W7*XNUS*KP**3-4*L*D48*
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XC14*P7Y0*W7*XNUS*KP
 FTOUT=FTOUT+2*L*D48*C14*P6Y0*W7*XNUS*KP-2*L*D48*C14*P5Y0*W7*XNUS*
XKP**3+4*L*D48*C14*P4Y0*W7*XNUS*KP-2*L*D48*C14*P3Y0*W7*XNUS*KP-2*L
X*D48*C14*W7*P8Y2*XNUS*KP**3+4*L*D48*C14*W7*P7Y2*XNUS*KP-2*L*D48*
XC14*W7*P6Y2*XNUS*KP+2*L*D48*C14*W7*XNUS*KP**3*P5Y2-4*L*D48*P4Y3*
XC13Y*W5-2*L*D48*P3Y1*C13Y*W5+2*L*D48*P3Y2*C13*W5+2*L*D48*P3Y3*
XC13Y*W5+2*L*D48*P2Y2*C4*W3*XNUS*KP-2*L*D48*C7*W5*XNUS**2*KP**2*
XP5Y1+2*L*D48*C7*W5*XNUS**2*KP**2*P5Y3-2*L*D48*C9*P5Y0*W5*XNUS**2*
XKP**2+2*L*D48*C9*P4Y0*W5+2*L*D48*C9*W5*XNUS**2*KP**2*P5Y2-2*L*D48
X*C13*P5Y0*W5*XNUS**2*KP**2+4*L*D48*C13*P4Y0*W5-2*L*D48*C13*P3Y0*
XW5
 FTOUT=FTOUT+2*L*D48*C13*W5*XNUS**2*KP**2*P5Y2-2*L*D48*C13Y*W5*XNUS
X**2*KP**2*P5Y1+2*L*D48*C13Y*W5*XNUS**2*KP**2*P5Y3-2*L*D48*C4*P2Y0
X*W3*XNUS*KP+2*L*C14*D43*P8Y0*W5*XNUS**2*KP**2-2*L*C14*D43*P7Y0*W5
X-2*L*C14*D43*W5*P8Y2*XNUS**2*KP**2+2*L*C14*D43*W5*P7Y2+2*L*C14*
XD53*W5*P8Y1*XNUS**2*KP**2-2*L*C14*D53*W5*P8Y3*XNUS**2*KP**2+2*L*
XC14*D43Y*W5*P8Y1*XNUS**2*KP**2-2*L*C14*D43Y*W5*P8Y3*XNUS**2*KP**2
X-2*L*C14*D43Y*W5*P7Y1+2*L*C14*D43Y*W5*P7Y3+2*L*C14*D53Y*W5*P8Y2*
XXNUS**2*KP**2-2*L*C14*D53Y*W5*P8Y4*XNUS**2*KP**2+2*L*P2Y1*C7*D43*
XW3*XNUS*KP+2*L*P2Y1*C9*D43Y*W3*XNUS*KP+2*L*P2Y1*D43*C16*W3*XNUS*
XKP
 FTOUT=FTOUT+2*L*P2Y1*D43*C13Y*W3*XNUS*KP+2*L*P2Y1*D53*C15*W3*XNUS*
XKP+2*L*P2Y1*D53*C13*W3*XNUS*KP+2*L*P2Y1*C16*D53Y*W3*XNUS*KP+2*L*
XP2Y1*C15*D43Y*W3*XNUS*KP+2*L*P2Y1*D43Y*C13*W3*XNUS*KP+2*L*P2Y2*C7
X*D43Y*W3*XNUS*KP-2*L*P2Y2*C9*D43*W3*XNUS*KP-2*L*P2Y2*D43*C15*W3*
XXNUS*KP-2*L*P2Y2*D43*C13*W3*XNUS*KP-2*L*P2Y2*D53*C16*W3*XNUS*KP+2
X*L*P2Y2*D53*C13Y*W3*XNUS*KP+2*L*P2Y2*C16*D43Y*W3*XNUS*KP+2*L*P2Y2
X*C15*D53Y*W3*XNUS*KP+2*L*P2Y2*D43Y*C13Y*W3*XNUS*KP+2*L*P2Y2*D53Y*
XC13*W3*XNUS*KP-2*L*P2Y3*C7*D43*W3*XNUS*KP-2*L*P2Y3*C9*D43Y*W3*
XXNUS*KP-2*L*P2Y3*D43*C16*W3*XNUS*KP-2*L*P2Y3*D43*C13Y*W3*XNUS*KP-
X2*L*P2Y3*D53*C15*W3*XNUS*KP
 FTOUT=FTOUT-2*L*P2Y3*D53*C13*W3*XNUS*KP-2*L*P2Y3*C16*D53Y*W3*XNUS*
XKP-2*L*P2Y3*C15*D43Y*W3*XNUS*KP-2*L*P2Y3*D43Y*C13*W3*XNUS*KP+2*L*
XP2Y3*D53Y*C13Y*W3*XNUS*KP-2*L*P2Y4*C7*D43Y*W3*XNUS*KP-2*L*P2Y4*
XD53*C13Y*W3*XNUS*KP-2*L*P2Y4*C16*D43Y*W3*XNUS*KP-2*L*P2Y4*C15*
XD53Y*W3*XNUS*KP-2*L*P2Y4*D43Y*C13Y*W3*XNUS*KP-2*L*P2Y4*D53Y*C13*
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     XW3*XNUS*KP-2*L*P2Y5*D53Y*C13Y*W3*XNUS*KP+2*L*C9*D43*P2Y0*W3*XNUS*
     XKP+2*L*D43*C15*P2Y0*W3*XNUS*KP+2*L*D43*C13*P2Y0*W3*XNUS*KP+2*L*
     XD53*C16*P2Y0*W3*XNUS*KP
C                           TERM4
      FTOUT=
     XFTOUT+4*L*W*C20*D43*P1Y2+4*L*W*C20*D53*P1Y1+4*L*W*C20*P1Y2*D53Y+4
     X*L*W*C20*D43Y*P1Y3+4*L*W*C20Y*D43*P1Y3+4*L*W*C20Y*D53*P1Y2+4*L*W*
     XC20Y*D43Y*P1Y4+4*L*W*C20Y*P1Y3*D53Y+2*L*W*C30*D53*P1Y1-2*L*W*C30*
     XD53*P1Y3+2*L*W*C30*P1Y2*D53Y-2*L*W*C30*P1Y4*D53Y-2*L*W*C31*D43*
     XP1Y1+2*L*W*C31*D43*P1Y3-2*L*W*C31*P1Y2*D43Y+2*L*W*C31*D43Y*P1Y4+2
     X*L*X**2*D48*C28*P5Y0*KP**2*W4+2*L*X**2*D48*C28*KP**2*W5*P5Y1+4*L*
     XX*D48*C28*XNUS*KP**2*W5*P5Y2+4*L*X*D48*C28*XNUS*KP**2*W4*P5Y1+2*L
     X*X*P2Y0*KP*D43*C29*W2-4*L*X*P2Y0*KP*D53*C22*W2-4*L*X*P2Y1*KP*D43*
     XC22*W2+2*L*X*P2Y1*KP*D43*C29*W3-4*L*X*P2Y1*KP*D53*C22*W3-4*L*X*
     XP2Y1*KP*D53*C24*W2
      FTOUT=FTOUT-4*L*X*P2Y1*KP*C22*D53Y*W2+2*L*X*P2Y1*KP*D43Y*C29*W2-4*
     XL*X*P2Y2*KP*D43*C22*W3-4*L*X*P2Y2*KP*D43*C24*W2-2*L*X*P2Y2*KP*D43
     X*C29*W2-4*L*X*P2Y2*KP*D53*C24*W3-4*L*X*P2Y2*KP*C22*D43Y*W2-4*L*X*
     XP2Y2*KP*C22*D53Y*W3-4*L*X*P2Y2*KP*C24*D53Y*W2+2*L*X*P2Y2*KP*D43Y*
     XC29*W3-4*L*X*P2Y3*KP*D43*C24*W3-2*L*X*P2Y3*KP*D43*C29*W3-4*L*X*
     XP2Y3*KP*C22*D43Y*W3-4*L*X*P2Y3*KP*C24*D43Y*W2-4*L*X*P2Y3*KP*C24*
     XD53Y*W3-2*L*X*P2Y3*KP*D43Y*C29*W2-4*L*X*P2Y4*KP*C24*D43Y*W3-2*L*X
     X*P2Y4*KP*D43Y*C29*W3-2*L*D48*C28*P4Y1*W5-2*L*D48*C28*P4Y0*W4+2*L*
     XD48*C28*P3Y0*W4+2*L*D48*C28*P3Y1*W5+2*L*D48*C28*XNUS**2*KP**2*W5*
     XP5Y3
      FTOUT=FTOUT+2*L*D48*C28*XNUS**2*KP**2*W4*P5Y2+2*L*D48*P2Y0*XNUS*KP
     X*C27*W2+2*L*D48*P2Y1*XNUS*KP*C27*W3-2*L*D48*P2Y2*XNUS*KP*C27*W2-2
     X*L*D48*P2Y3*XNUS*KP*C27*W3-2*L*P2Y0*XNUS*KP*D53*C38*W2-4*L*P2Y0*
     XXNUS*KP*D53*C24*W2-2*L*P2Y0*XNUS*KP*D53*C29*W2+4*L*P1Y0*C20*D53+2
     X*L*P1Y0*C30*D53-2*L*P1Y0*C31*D43+4*L*C20*D43*P1Y1+4*L*C20*P1Y1*
     XD53Y+4*L*C20*P1Y2*D43Y+4*L*C20Y*D43*P1Y2+4*L*C20Y*D53*P1Y1+4*L*
     XC20Y*P1Y2*D53Y+4*L*C20Y*D43Y*P1Y3-2*L*C30*D53*P1Y2+2*L*C30*P1Y1*
     XD53Y-2*L*C30*P1Y3*D53Y+2*L*C31*D43*P1Y2-2*L*C31*P1Y1*D43Y+2*L*C31
     X*D43Y*P1Y3-4*L*P2Y1*XNUS*KP*D43*C24*W2-2*L*P2Y1*XNUS*KP*D53*C38*
     XW3
      FTOUT=FTOUT-4*L*P2Y1*XNUS*KP*D53*C22*W2-4*L*P2Y1*XNUS*KP*D53*C24*
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XW3-2*L*P2Y1*XNUS*KP*D53*C29*W3-2*L*P2Y1*XNUS*KP*C38*D53Y*W2-4*L*
XP2Y1*XNUS*KP*C24*D53Y*W2-2*L*P2Y1*XNUS*KP*D53Y*C29*W2-4*L*P2Y2*
XXNUS*KP*D43*C22*W2-4*L*P2Y2*XNUS*KP*D43*C24*W3+2*L*P2Y2*XNUS*KP*
XD53*C38*W2-4*L*P2Y2*XNUS*KP*D53*C22*W3+2*L*P2Y2*XNUS*KP*D53*C29*
XW2-2*L*P2Y2*XNUS*KP*C38*D53Y*W3-4*L*P2Y2*XNUS*KP*C22*D53Y*W2-4*L*
XP2Y2*XNUS*KP*C24*D43Y*W2-4*L*P2Y2*XNUS*KP*C24*D53Y*W3-2*L*P2Y2*
XXNUS*KP*D53Y*C29*W3-4*L*P2Y3*XNUS*KP*D43*C22*W3+2*L*P2Y3*XNUS*KP*
XD53*C38*W3+2*L*P2Y3*XNUS*KP*D53*C29*W3+2*L*P2Y3*XNUS*KP*C38*D53Y*
XW2
 FTOUT=FTOUT-4*L*P2Y3*XNUS*KP*C22*D43Y*W2-4*L*P2Y3*XNUS*KP*C22*D53Y
X*W3-4*L*P2Y3*XNUS*KP*C24*D43Y*W3+2*L*P2Y3*XNUS*KP*D53Y*C29*W2+2*L
X*XNUS*P2Y4*KP*C38*D53Y*W3-4*L*XNUS*P2Y4*KP*C22*D43Y*W3+2*L*XNUS*
XP2Y4*KP*D53Y*C29*W3+4*W*C53*D43*P1Y2+4*W*C53*D53*P1Y1+4*W*C53*
XP1Y2*D53Y+4*W*C53*D43Y*P1Y3+4*W*C43*D43*P1Y1+4*W*C43*D53*P1Y2+4*W
X*C43*P1Y2*D43Y+4*W*C43*P1Y3*D53Y+4*W*C53Y*D43*P1Y3+4*W*C53Y*D53*
XP1Y2+4*W*C53Y*D43Y*P1Y4+4*W*C53Y*P1Y3*D53Y+4*W*C43Y*D43*P1Y2+4*W*
XC43Y*D53*P1Y3+4*W*C43Y*D43Y*P1Y3+4*W*C43Y*P1Y4*D53Y+2*W*C42*D43*
XP1Y1-2*W*C42*D43*P1Y3+2*W*C42*P1Y2*D43Y-2*W*C42*D43Y*P1Y4+2*X**2*
XD48*C44*P5Y0*KP**2*W4
 FTOUT=FTOUT+2*X**2*D48*C44*KP**2*W5*P5Y1+6*X**2*D48*P5Y0*C48*KP**2
X*W4+2*X**2*D48*P5Y0*KP**2*C40*W4+6*X**2*D48*C48*KP**2*W5*P5Y1+2*X
X**2*D48*KP**2*C40*W5*P5Y1+2*X**2*D48*KP**2*C40Y*W5*P5Y2+2*X**2*
XD48*KP**2*C40Y*W4*P5Y1+4*X*D48*C44*XNUS*KP**2*W5*P5Y2+4*X*D48*C44
X*XNUS*KP**2*W4*P5Y1+12*X*D48*C48*XNUS*KP**2*W5*P5Y2+12*X*D48*C48*
XXNUS*KP**2*W4*P5Y1+4*X*D48*XNUS*KP**2*C40*W5*P5Y2+4*X*D48*XNUS*KP
X**2*C40*W4*P5Y1+4*X*D48*XNUS*KP**2*C40Y*W5*P5Y3+4*X*D48*XNUS*KP**
X2*C40Y*W4*P5Y2-4*X*P2Y0*KP*C40*D53*W2-4*X*P2Y1*KP*D43*C40*W2-4*X*
XP2Y1*KP*C40*D53*W3-4*X*P2Y1*KP*C40*D53Y*W2-4*X*P2Y1*KP*D53*C40Y*
XW2
 FTOUT=FTOUT-4*X*P2Y2*KP*D43*C40*W3-4*X*P2Y2*KP*D43*C40Y*W2-4*X*
XP2Y2*KP*C40*D43Y*W2-4*X*P2Y2*KP*C40*D53Y*W3-4*X*P2Y2*KP*D53*C40Y*
XW3-4*X*P2Y2*KP*C40Y*D53Y*W2-4*X*P2Y3*KP*D43*C40Y*W3-4*X*P2Y3*KP*
XC40*D43Y*W3-4*X*P2Y3*KP*D43Y*C40Y*W2-4*X*P2Y3*KP*C40Y*D53Y*W3-4*X
X*P2Y4*KP*D43Y*C40Y*W3-2*D48*C44*P4Y1*W5-2*D48*C44*P4Y0*W4+2*D48*
XC44*P3Y0*W4+2*D48*C44*P3Y1*W5+2*D48*C44*XNUS**2*KP**2*W5*P5Y3+2*
XD48*C44*XNUS**2*KP**2*W4*P5Y2-6*D48*P4Y1*C48*W5-2*D48*P4Y1*C40*W5
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      X-2*D48*P4Y1*C40Y*W4-2*D48*P4Y2*C40Y*W5-6*D48*P4Y0*C48*W4-2*D48*
      XP4Y0*C40*W4+6*D48*P3Y0*C48*W4+2*D48*P3Y0*C40*W4-2*D48*P2Y0*C43*
      XXNUS*KP*W2
       FTOUT=FTOUT-6*D48*P2Y0*C42*XNUS*KP*W2+6*D48*C48*P3Y1*W5+6*D48*C48*
      XXNUS**2*KP**2*W5*P5Y3+6*D48*C48*XNUS**2*KP**2*W4*P5Y2-2*D48*C43*
      XP2Y1*XNUS*KP*W3+2*D48*C43*P2Y2*XNUS*KP*W2+2*D48*C43*P2Y3*XNUS*KP*
      XW3+2*D48*P3Y1*C40*W5+2*D48*P3Y1*C40Y*W4+2*D48*P3Y2*C40Y*W5-2*D48*
      XC43Y*P2Y1*XNUS*KP*W2-2*D48*C43Y*P2Y2*XNUS*KP*W3+2*D48*C43Y*P2Y3*
      XXNUS*KP*W2+2*D48*C43Y*XNUS*P2Y4*KP*W3-6*D48*C42*P2Y1*XNUS*KP*W3+6
      X*D48*C42*P2Y2*XNUS*KP*W2+6*D48*C42*P2Y3*XNUS*KP*W3+2*D48*XNUS**2*
      XKP**2*C40*W5*P5Y3+2*D48*XNUS**2*KP**2*C40*W4*P5Y2+2*D48*XNUS**2*
      XKP**2*C40Y*W5*P5Y4+2*D48*XNUS**2*KP**2*C40Y*W4*P5Y3-2*C44*P2Y0*
      XXNUS*KP*D43*W2
       FTOUT=FTOUT-2*C44*P2Y1*XNUS*KP*D43*W3-2*C44*P2Y1*XNUS*KP*D43Y*W2+2
      X*C44*P2Y2*XNUS*KP*D43*W2-2*C44*P2Y2*XNUS*KP*D43Y*W3+2*C44*P2Y3*
      XXNUS*KP*D43*W3+2*C44*P2Y3*XNUS*KP*D43Y*W2+2*C44*XNUS*P2Y4*KP*D43Y
      X*W3-2*P2Y0*C48*XNUS*KP*D43*W2-4*P2Y0*XNUS*KP*D43*C40*W2+4*P1Y0*
      XC53*D53+4*P1Y0*C43*D43+2*P1Y0*C42*D43-2*C48*P2Y1*XNUS*KP*D43*W3-2
      X*C48*P2Y1*XNUS*KP*D43Y*W2+2*C48*P2Y2*XNUS*KP*D43*W2-2*C48*P2Y2*
      XXNUS*KP*D43Y*W3+2*C48*P2Y3*XNUS*KP*D43*W3+2*C48*P2Y3*XNUS*KP*D43Y
      X*W2+2*C48*XNUS*P2Y4*KP*D43Y*W3+4*C53*D43*P1Y1+4*C53*P1Y1*D53Y+4*
      XC53*P1Y2*D43Y+4*C43*D53*P1Y1+4*C43*P1Y1*D43Y+4*C43*P1Y2*D53Y+4*
      XC53Y*D43*P1Y2
       FTOUT=FTOUT+4*C53Y*D53*P1Y1+4*C53Y*P1Y2*D53Y+4*C53Y*D43Y*P1Y3+4*
      XC43Y*D43*P1Y1+4*C43Y*D53*P1Y2+4*C43Y*P1Y2*D43Y+4*C43Y*P1Y3*D53Y-2
      X*C42*D43*P1Y2+2*C42*P1Y1*D43Y-2*C42*D43Y*P1Y3-4*P2Y1*XNUS*KP*D43*
      XC40*W3-4*P2Y1*XNUS*KP*D43*C40Y*W2-4*P2Y1*XNUS*KP*C40*D53*W2-4*
      XP2Y1*XNUS*KP*C40*D43Y*W2-4*P2Y2*XNUS*KP*D43*C40Y*W3-4*P2Y2*XNUS*
      XKP*C40*D53*W3-4*P2Y2*XNUS*KP*C40*D43Y*W3-4*P2Y2*XNUS*KP*C40*D53Y*
      XW2-4*P2Y2*XNUS*KP*D53*C40Y*W2-4*P2Y2*XNUS*KP*D43Y*C40Y*W2-4*P2Y3*
      XXNUS*KP*C40*D53Y*W3-4*P2Y3*XNUS*KP*D53*C40Y*W3-4*P2Y3*XNUS*KP*
      XD43Y*C40Y*W3-4*P2Y3*XNUS*KP*C40Y*D53Y*W2-4*XNUS*P2Y4*KP*C40Y*D53Y
      X*W3
C                         TERM5
       FTOUT=
      XFTOUT+2*LAW*C20*D43*P1Y0-2*LAW*C20*D43*P1Y2+2*LAW*C20*D43Y*P1Y1-2
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X*L*W*C20*D43Y*P1Y3+2*L*W*C20Y*D43*P1Y1-2*L*W*C20Y*D43*P1Y3+2*L*W*
XC20Y*D43Y*P1Y2-2*L*W*C20Y*D43Y*P1Y4+2*L*W*D53*C27*P1Y0-2*L*W*D53*
XC27*P1Y2+2*L*W*D53Y*C27*P1Y1-2*L*W*D53Y*C27*P1Y3+2*L*X**2*C29*D48
X*P5Y2*KP**2*W5-2*L*X**2*C29*D48*P5Y0*KP**2*W5-2*L*X**2*D48*P5Y1*
XC28*KP**2*W5-4*L*X*C29*D48*P5Y1*XNUS*KP**2*W5+4*L*X*C29*D48*P5Y3*
XXNUS*KP**2*W5-2*L*X*D48*P5Y1*C24*XNUS*KP**2*W5+2*L*X*D48*P5Y2*C22
X*XNUS*KP**2*W5-2*L*X*D48*P5Y2*C28*XNUS*KP**2*W5+2*L*X*D48*P5Y3*
XC24*XNUS*KP**2*W5-2*L*X*D48*C22*P5Y0*XNUS*KP**2*W5-2*L*X*D48*P5Y0
X*C28*XNUS*KP**2*W5
 FTOUT=FTOUT-2*L*X*D43*C22*P2Y0*KP*W3+2*L*X*D43*C22*P2Y2*KP*W3-2*L*
XX*D43*C24*P2Y1*KP*W3+2*L*X*D43*C24*P2Y3*KP*W3-2*L*X*D43*C28*P2Y0*
XKP*W3+2*L*X*D43*C28*P2Y2*KP*W3-2*L*X*D43Y*C22*P2Y1*KP*W3+2*L*X*
XD43Y*C22*P2Y3*KP*W3-2*L*X*D43Y*C24*P2Y2*KP*W3+2*L*X*D43Y*C24*P2Y4
X*KP*W3-2*L*X*D43Y*C28*P2Y1*KP*W3+2*L*X*D43Y*C28*P2Y3*KP*W3+2*L*
XC29*D48*P5Y2*XNUS**2*KP**2*W5-4*L*C29*D48*P4Y2*W5+2*L*C29*D48*
XP3Y2*W5-2*L*C29*D48*P5Y0*XNUS**2*KP**2*W5+4*L*C29*D48*P4Y0*W5-2*L
X*C29*D48*P3Y0*W5-2*L*D48*C20*P2Y0*XNUS*KP*W3+2*L*D48*C20*P2Y2*
XXNUS*KP*W3-2*L*D48*C20Y*P2Y1*XNUS*KP*W3+2*L*D48*C20Y*P2Y3*XNUS*KP
X*W3
 FTOUT=FTOUT-2*L*D48*P5Y1*C22*XNUS**2*KP**2*W5-2*L*D48*P5Y1*C28*
XXNUS**2*KP**2*W5+2*L*D48*P5Y2*C24*XNUS**2*KP**2*W5+2*L*D48*P5Y3*
XC22*XNUS**2*KP**2*W5+2*L*D48*P4Y1*C28*W5-2*L*D48*P4Y2*C24*W5-2*L*
XD48*P3Y1*C28*W5-2*L*D48*C24*P5Y0*XNUS**2*KP**2*W5+2*L*D48*C24*
XP4Y0*W5+2*L*D53*C28*P2Y0*XNUS*KP*W3-2*L*D53*C28*P2Y2*XNUS*KP*W3-2
X*L*D43*C22*P2Y1*XNUS*KP*W3+2*L*D43*C22*P2Y3*XNUS*KP*W3-2*L*D43*
XC24*P2Y0*XNUS*KP*W3+2*L*D43*C24*P2Y2*XNUS*KP*W3+2*L*D53Y*C28*P2Y1
X*XNUS*KP*W3-2*L*D53Y*C28*P2Y3*XNUS*KP*W3-2*L*D43Y*C22*P2Y2*XNUS*
XKP*W3+2*L*D43Y*C22*P2Y4*XNUS*KP*W3-2*L*D43Y*C24*P2Y1*XNUS*KP*W3+2
X*L*D43Y*C24*P2Y3*XNUS*KP*W3
 FTOUT=FTOUT-2*W*C42*D53*P1Y0+2*W*C42*D53*P1Y2-2*W*C42*D43*P1Y1+2*W
X*C42*D43*P1Y3-2*W*C42*D53Y*P1Y1+2*W*C42*D53Y*P1Y3-2*W*C42*D43Y*
XP1Y2+2*W*C42*D43Y*P1Y4-2*W*D43*P1Y0*C41+2*W*D43*C41*P1Y2-2*W*D43Y
X*C41*P1Y1+2*W*D43Y*C41*P1Y3-2*X**2*D48*P5Y1*C48*KP**2*W5-2*X*D48*
XC44*P5Y2*XNUS*KP**2*W5+2*X*D48*C44*P5Y0*XNUS*KP**2*W5-4*X*D48*
XP5Y2*C48*XNUS*KP**2*W5+2*X*C44*D43*P2Y0*KP*W3-2*X*C44*D43*P2Y2*KP
X*W3+2*X*C44*D43Y*P2Y1*KP*W3-2*X*C44*D43Y*P2Y3*KP*W3+2*D48*C42*
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XP2Y1*XNUS*KP*W3-2*D48*C42*P2Y3*XNUS*KP*W3+2*D48*C44*P5Y1*XNUS**2*
XKP**2*W5-2*D48*C44*P5Y3*XNUS**2*KP**2*W5-2*D48*P5Y3*C48*XNUS**2*
XKP**2*W5
 FTOUT=FTOUT+2*D48*C48*P4Y1*W5-2*D48*C48*P3Y1*W5+2*D48*P2Y0*C41*
XXNUS*KP*W3-2*D48*C41*P2Y2*XNUS*KP*W3+2*C44*D43*P2Y1*XNUS*KP*W3-2*
XC44*D43*P2Y3*XNUS*KP*W3+2*C44*D43Y*P2Y2*XNUS*KP*W3-2*C44*D43Y*
XP2Y4*XNUS*KP*W3+2*D53*C48*P2Y0*XNUS*KP*W3-2*D53*C48*P2Y2*XNUS*KP*
XW3+2*C48*D43*P2Y1*XNUS*KP*W3-2*C48*D43*P2Y3*XNUS*KP*W3+2*C48*D53Y
X*P2Y1*XNUS*KP*W3-2*C48*D53Y*P2Y3*XNUS*KP*W3+2*C48*D43Y*P2Y2*XNUS*
XKP*W3-2*C48*D43Y*P2Y4*XNUS*KP*W3
 FTOUT=
XFTOUT+2*L*X**2*C29*D48*W5*P5Y0*KP**2-2*L*X**2*C29*D48*W5*P5Y2*KP
X**2-2*L*X**2*D48*W5*C28*P5Y1*KP**2+4*L*X*C29*D48*W5*P5Y1*XNUS*KP
X**2-4*L*X*C29*D48*W5*P5Y3*XNUS*KP**2+2*L*X*D48*W5*C22*P5Y0*XNUS*
XKP**2-2*L*X*D48*W5*C22*P5Y2*XNUS*KP**2+2*L*X*D48*W5*C24*P5Y1*XNUS
X*KP**2-2*L*X*D48*W5*C24*P5Y3*XNUS*KP**2-4*L*X*D48*W5*C28*P5Y2*
XXNUS*KP**2+2*L*C29*D48*W5*P5Y0*XNUS**2*KP**2-4*L*C29*D48*W5*P4Y0+
X2*L*C29*D48*W5*P3Y0-2*L*C29*D48*W5*P5Y2*XNUS**2*KP**2+4*L*C29*D48
X*W5*P4Y2-2*L*C29*D48*W5*P3Y2+2*L*D48*C20*W3*P2Y0*XNUS*KP-2*L*D48*
XC20*W3*XNUS*KP*P2Y2-2*L*D48*C27*W3*XNUS*KP*P2Y1+2*L*D48*C27*W3*
XXNUS*KP*P2Y3
  FTOUT=FTOUT+2*L*D48*C20Y*W3*XNUS*KP*P2Y1-2*L*D48*C20Y*W3*XNUS*KP*
XP2Y3+2*L*D48*W5*C22*P5Y1*XNUS**2*KP**2-2*L*D48*W5*C22*P5Y3*XNUS**
X2*KP**2+2*L*D48*W5*C24*P5Y0*XNUS**2*KP**2-2*L*D48*W5*C24*P4Y0-2*L
X*D48*W5*C24*P5Y2*XNUS**2*KP**2+2*L*D48*W5*C24*P4Y2-2*L*D48*W5*C28
X*P5Y3*XNUS**2*KP**2+2*L*D48*W5*C28*P4Y1-2*L*D48*W5*C28*P3Y1-2*X**
X2*D48*C48*W5*P5Y1*KP**2-4*X*D48*C48*W5*P5Y2*XNUS*KP**2+2*X*D48*W5
X*C40*P5Y0*XNUS*KP**2-2*X*D48*W5*C40*P5Y2*XNUS*KP**2+2*X*D48*W5*
XC40Y*P5Y1*XNUS*KP**2-2*X*D48*W5*C40Y*P5Y3*XNUS*KP**2-2*D48*C48*W5
X*P5Y3*XNUS**2*KP**2+2*D48*C48*W5*P4Y1-2*D48*C48*W5*P3Y1+2*D48*W5*
XC40*P5Y1*XNUS**2*KP**2
  FTOUT=FTOUT-2*D48*W5*C40*P5Y3*XNUS**2*KP**2+2*D48*W5*C40Y*P5Y2*
XXNUS**2*KP**2-2*D48*W5*C40Y*P5Y4*XNUS**2*KP**2+2*D48*C53*W3*P2Y0*
XXNUS*KP-2*D48*C53*W3*XNUS*KP*P2Y2+2*D48*W3*C42*XNUS*KP*P2Y1-2*D48
X*W3*C42*XNUS*KP*P2Y3+2*D48*W3*C43*XNUS*KP*P2Y1-2*D48*W3*C43*XNUS*
XKP*P2Y3+2*D48*W3*C53Y*XNUS*KP*P2Y1-2*D48*W3*C53Y*XNUS*KP*P2Y3+2*
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      XD48*W3*C43Y*XNUS*KP*P2Y2-2*D48*W3*C43Y*XNUS*KP*P2Y4
C                          TERM6
       FTOUT=
      XFTOUT+4*L*W*P1Y1*C53*D43+4*L*W*P1Y1*C53*D53Y+4*L*W*P1Y1*C43*D53+4
      X*L*W*P1Y1*C43*D43Y+2*L*W*P1Y1*C42*D43Y+4*L*W*P1Y1*C53Y*D53+4*L*W*
      XP1Y1*C43Y*D43+4*L*W*P1Y2*C53*D43Y+4*L*W*P1Y2*C43*D53Y-2*L*W*P1Y2*
      XC42*D43+4*L*W*P1Y2*C53Y*D43+4*L*W*P1Y2*C53Y*D53Y+4*L*W*P1Y2*C43Y*
      XD53+4*L*W*P1Y2*C43Y*D43Y-2*L*W*P1Y3*C42*D43Y+4*L*W*P1Y3*C53Y*D43Y
      X+4*L*W*P1Y3*C43Y*D53Y+4*L*W*C53*D53*P1Y0+4*L*W*C43*D43*P1Y0+2*L*W
      X*C42*D43*P1Y0-2*L*X**2*C44*D48*W5*P5Y0*KP**2-2*L*X**2*C44*D48*W4*
      XP5Y1*KP**2+2*L*X**2*D48*W5*C40*P5Y0*KP**2+2*L*X**2*D48*W5*C40Y*
      XP5Y1*KP**2+2*L*X**2*D48*W4*C40*P5Y1*KP**2+2*L*X**2*D48*W4*C40Y*
      XP5Y2*KP**2
       FTOUT=FTOUT-4*L*X*C44*D48*W5*P5Y1*XNUS*KP**2-2*L*X*C44*D48*W4*P5Y2
      X*XNUS*KP**2-2*L*X*C44*D48*W4*P5Y0*XNUS*KP**2+2*L*X*C44*W2*D43*
      XP2Y0*KP-2*L*X*C44*W2*D43*P2Y2*KP+2*L*X*C44*W2*D43Y*P2Y1*KP-2*L*X*
      XC44*W2*D43Y*P2Y3*KP+4*L*X*D48*W5*C40*P5Y1*XNUS*KP**2+4*L*X*D48*W5
      X*C40Y*P5Y2*XNUS*KP**2+2*L*X*D48*W4*C40*P5Y2*XNUS*KP**2+2*L*X*D48*
      XW4*C40*P5Y0*XNUS*KP**2+2*L*X*D48*W4*C40Y*P5Y1*XNUS*KP**2+2*L*X*
      XD48*W4*C40Y*P5Y3*XNUS*KP**2-4*L*X*D53*W3*C40*P2Y0*KP-4*L*X*D53*W3
      X*C40Y*P2Y1*KP-4*L*X*D53*W2*C40*P2Y1*KP-4*L*X*D53*W2*C40Y*P2Y2*KP-
      X4*L*X*W3*D43*C40*P2Y1*KP-4*L*X*W3*D43*C40Y*P2Y2*KP-4*L*X*W3*C40*
      XD53Y*P2Y1*KP
       FTOUT=FTOUT-4*L*X*W3*C40*D43Y*P2Y2*KP-4*L*X*W3*C40Y*D53Y*P2Y2*KP-4
      X*L*X*W3*C40Y*D43Y*P2Y3*KP-4*L*X*W2*D43*C40*P2Y2*KP-4*L*X*W2*D43*
      XC40Y*P2Y3*KP-4*L*X*W2*C40*D53Y*P2Y2*KP-4*L*X*W2*C40*D43Y*P2Y3*KP-
      X4*L*X*W2*C40Y*D53Y*P2Y3*KP-4*L*X*W2*C40Y*D43Y*P2Y4*KP-2*L*C44*D48
      X*W5*P5Y2*XNUS**2*KP**2+2*L*C44*D48*W5*P4Y0-2*L*C44*D48*W5*P3Y0-2*
      XL*C44*D48*W4*P5Y1*XNUS**2*KP**2+2*L*C44*D48*W4*P4Y1-2*L*C44*D48*
      XW4*P3Y1-2*L*C44*W3*D43*P2Y0*XNUS*KP+2*L*C44*W3*D43*P2Y2*XNUS*KP-2
      X*L*C44*W3*D43Y*P2Y1*XNUS*KP+2*L*C44*W3*D43Y*P2Y3*XNUS*KP+2*L*D48*
      XC53*W2*P2Y0*XNUS*KP-2*L*D48*C53*W2*P2Y2*XNUS*KP-2*L*D48*C43*W3*
      XP2Y0*XNUS*KP
       FTOUT=FTOUT+2*L*D48*C43*W3*P2Y2*XNUS*KP+2*L*D48*C53Y*W2*P2Y1*XNUS*
      XKP-2*L*D48*C53Y*W2*P2Y3*XNUS*KP-2*L*D48*C43Y*W3*P2Y1*XNUS*KP+2*L*
      XD48*C43Y*W3*P2Y3*XNUS*KP-2*L*D48*C41*W2*P2Y0*XNUS*KP+2*L*D48*C41*
```

```
XW2*P2Y2*XNUS*KP+2*L*D48*W5*C40*P5Y2*XNUS**2*KP**2-2*L*D48*W5*C40*
XP4Y0+2*L*D48*W5*C40*P3Y0+2*L*D48*W5*C40Y*P5Y3*XNUS**2*KP**2-2*L*
XD48*W5*C40Y*P4Y1+2*L*D48*W5*C40Y*P3Y1+2*L*D48*W4*C40*P5Y1*XNUS**2
X*KP**2-2*L*D48*W4*C40*P4Y1+2*L*D48*W4*C40*P3Y1+2*L*D48*W4*C40Y*
XP5Y2*XNUS**2*KP**2-2*L*D48*W4*C40Y*P4Y2+2*L*D48*W4*C40Y*P3Y2+4*L*
XP1Y1*C53*D53+4*L*P1Y1*C43*D43-2*L*P1Y1*C42*D53Y-2*L*P1Y1*C41*D43Y
X+4*L*P1Y2*C53*D43
 FTOUT=FTOUT+4*L*P1Y2*C53*D53Y+4*L*P1Y2*C43*D53+4*L*P1Y2*C43*D43Y+2
X*L*P1Y2*C42*D53+4*L*P1Y2*C53Y*D53+4*L*P1Y2*C43Y*D43+2*L*P1Y2*C41*
XD43+4*L*P1Y3*C53*D43Y+4*L*P1Y3*C43*D53Y+2*L*P1Y3*C42*D53Y+4*L*
XP1Y3*C53Y*D43+4*L*P1Y3*C53Y*D53Y+4*L*P1Y3*C43Y*D53+4*L*P1Y3*C43Y*
XD43Y+2*L*P1Y3*C41*D43Y+4*L*P1Y4*C53Y*D43Y+4*L*P1Y4*C43Y*D53Y-2*L*
XC42*D53*P1Y0-2*L*C41*D43*P1Y0-4*L*D53*W3*C40*P2Y1*XNUS*KP-4*L*D53
X*W3*C40Y*P2Y2*XNUS*KP-4*L*D53*W2*C40*P2Y2*XNUS*KP-4*L*D53*W2*C40Y
X*P2Y3*XNUS*KP+2*L*D53*W2*C48*P2Y0*XNUS*KP-2*L*D53*W2*C48*P2Y2*
XXNUS*KP-4*L*W3*D43*C40*P2Y0*XNUS*KP-4*L*W3*D43*C40Y*P2Y1*XNUS*KP-
X2*L*W3*D43*C48*P2Y0*XNUS*KP
 FTOUT=FTOUT+2*L*W3*D43*C48*P2Y2*XNUS*KP-4*L*W3*C40*D53Y*P2Y2*XNUS*
XKP-4*L*W3*C40*D43Y*P2Y1*XNUS*KP-4*L*W3*C40Y*D53Y*P2Y3*XNUS*KP-4*L
X*W3*C40Y*D43Y*P2Y2*XNUS*KP-2*L*W3*D43Y*C48*P2Y1*XNUS*KP+2*L*W3*
XD43Y*C48*P2Y3*XNUS*KP-4*L*W2*D43*C40*P2Y1*XNUS*KP-4*L*W2*D43*C40Y
X*P2Y2*XNUS*KP-4*L*W2*C40*D53Y*P2Y3*XNUS*KP-4*L*W2*C40*D43Y*P2Y2*
XXNUS*KP-4*L*W2*C40Y*D53Y*P2Y4*XNUS*KP-4*L*W2*C40Y*D43Y*P2Y3*XNUS*
XKP+2*L*W2*D53Y*C48*P2Y1*XNUS*KP-2*L*W2*D53Y*C48*P2Y3*XNUS*KP+4*W*
XC20*P1Y1*D43+4*W*C20*P1Y1*D53Y+4*W*C20*P1Y2*D43Y+4*W*C20*D53*P1Y0
X+2*W*C30*P1Y1*D53Y-2*W*C30*P1Y2*D53-2*W*C30*P1Y3*D53Y+2*W*C30*D53
X*P1Y0
 FTOUT=FTOUT-2*W*C31*P1Y1*D43Y+2*W*C31*P1Y2*D43+2*W*C31*P1Y3*D43Y-2
X*W*C31*D43*P1Y0+4*W*P1Y1*D53*C20Y+4*W*P1Y2*D43*C20Y+4*W*P1Y2*C20Y
X*D53Y+4*W*P1Y3*C20Y*D43Y-4*X**2*D48*C38*W5*P5Y1*KP**2-4*X**2*D48*
XC38*W4*P5Y0*KP**2-8*X*D48*C38*W5*P5Y2*XNUS*KP**2-8*X*D48*C38*W4*
XP5Y1*XNUS*KP**2-4*X*D48*C22*W4*P5Y2*XNUS*KP**2+4*X*D48*C22*W4*
XP5Y0*XNUS*KP**2-2*X*D48*C28*W4*P5Y2*XNUS*KP**2+2*X*D48*C28*W4*
XP5Y0*XNUS*KP**2+4*X*D48*C24*W4*P5Y1*XNUS*KP**2-4*X*D48*C24*W4*
XP5Y3*XNUS*KP**2-4*X*D48*W5*C29*P5Y2*XNUS*KP**2+4*X*D48*W5*C29*
XP5Y0*XNUS*KP**2+4*X*D48*W4*C29*P5Y1*XNUS*KP**2-4*X*D48*W4*C29*
```

```
XP5Y3*XNUS*KP**2
 FTOUT=FTOUT-4*X*C22*D53*W3*P2Y0*KP-4*X*C22*D53*W2*P2Y1*KP-4*X*C22*
XW3*D43*P2Y1*KP-4*X*C22*W3*D53Y*P2Y1*KP-4*X*C22*W3*D43Y*P2Y2*KP-2*
XX*C22*W2*D43*P2Y0*KP-2*X*C22*W2*D43*P2Y2*KP-4*X*C22*W2*D53Y*P2Y2*
XKP-2*X*C22*W2*D43Y*P2Y1*KP-2*X*C22*W2*D43Y*P2Y3*KP-2*X*C28*W2*D43
X*P2Y0*KP+2*X*C28*W2*D43*P2Y2*KP-2*X*C28*W2*D43Y*P2Y1*KP+2*X*C28*
XW2*D43Y*P2Y3*KP-4*X*C24*D53*W3*P2Y1*KP-4*X*C24*D53*W2*P2Y2*KP-4*X
X*C24*W3*D43*P2Y2*KP-4*X*C24*W3*D53Y*P2Y2*KP-4*X*C24*W3*D43Y*P2Y3*
XKP-2*X*C24*W2*D43*P2Y1*KP-2*X*C24*W2*D43*P2Y3*KP-4*X*C24*W2*D53Y*
XP2Y3*KP-2*X*C24*W2*D43Y*P2Y2*KP-2*X*C24*W2*D43Y*P2Y4*KP+2*X*W3*
XC29*D43*P2Y0*KP
 FTOUT=FTOUT-2*X*W3*C29*D43*P2Y2*KP+2*X*W3*C29*D43Y*P2Y1*KP-2*X*W3*
XC29*D43Y*P2Y3*KP+2*X*W2*C29*D43*P2Y1*KP-2*X*W2*C29*D43*P2Y3*KP+2*
XX*W2*C29*D43Y*P2Y2*KP-2*X*W2*C29*D43Y*P2Y4*KP+4*C20*D48*W2*P2Y0*
XXNUS*KP-4*C20*D48*W2*P2Y2*XNUS*KP+4*C20*P1Y1*D53+2*C20*P1Y1*D43Y+
X2*C20*P1Y2*D43+4*C20*P1Y2*D53Y+2*C20*P1Y3*D43Y+2*C20*D43*P1Y0+4*
XD48*C30*W3*P2Y1*XNUS*KP-4*D48*C30*W3*P2Y3*XNUS*KP+4*D48*C30*W2*
XP2Y0*XNUS*KP-4*D48*C30*W2*P2Y2*XNUS*KP+4*D48*C31*W3*P2Y0*XNUS*KP-
X4*D48*C31*W3*P2Y2*XNUS*KP+4*D48*C31*W2*P2Y1*XNUS*KP-4*D48*C31*W2*
XP2Y3*XNUS*KP-4*D48*C38*W5*P5Y3*XNUS**2*KP**2+4*D48*C38*W5*P4Y1-4*
XD48*C38*W5*P3Y1
 FTOUT=FTOUT-4*D48*C38*W4*P5Y2*XNUS**2*KP**2+4*D48*C38*W4*P4Y0-4*
XD48*C38*W4*P3Y0+4*D48*C22*W4*P5Y1*XNUS**2*KP**2-4*D48*C22*W4*P5Y3
X*XNUS**2*KP**2+2*D48*C28*W4*P5Y1*XNUS**2*KP**2-2*D48*C28*W4*P5Y3*
XXNUS**2*KP**2-4*D48*C24*W4*P5Y2*XNUS**2*KP**2+4*D48*C24*W4*P5Y0*
XXNUS**2*KP**2-4*D48*C24*W4*P4Y0+4*D48*C24*W4*P4Y2+4*D48*W5*C29*
XP5Y1*XNUS**2*KP**2-4*D48*W5*C29*P5Y3*XNUS**2*KP**2-4*D48*W4*C29*
XP5Y2*XNUS**2*KP**2+4*D48*W4*C29*P5Y0*XNUS**2*KP**2-4*D48*W4*C29*
XP4Y0+4*D48*W4*C29*P4Y2+4*D48*W2*C20Y*P2Y1*XNUS*KP-4*D48*W2*C20Y*
XP2Y3*XNUS*KP-2*D48*W2*C27*P2Y1*XNUS*KP+2*D48*W2*C27*P2Y3*XNUS*KP+
X2*C30*P1Y1*D53
 FTOUT=FTOUT+2*C30*P1Y2*D53Y-2*C30*P1Y3*D53-2*C30*P1Y4*D53Y-2*C31*
XP1Y1*D43-2*C31*P1Y2*D43Y+2*C31*P1Y3*D43+2*C31*P1Y4*D43Y+2*P1Y1*
XD43*C20Y+2*P1Y1*D53Y*C27+4*P1Y2*D53*C20Y-2*P1Y2*D53*C27+2*P1Y2*
XC20Y*D43Y+2*P1Y3*D43*C20Y+4*P1Y3*C20Y*D53Y-2*P1Y3*D53Y*C27+2*P1Y4
X*C20Y*D43Y-2*C38*D53*W3*P2Y0*XNUS*KP+2*C38*D53*W3*P2Y2*XNUS*KP-2*
```

```
XC38*D53*W2*P2Y1*XNUS*KP+2*C38*D53*W2*P2Y3*XNUS*KP-2*C38*W3*D53Y*
XP2Y1*XNUS*KP+2*C38*W3*D53Y*P2Y3*XNUS*KP-2*C38*W2*D53Y*P2Y2*XNUS*
XKP+2*C38*W2*D53Y*P2Y4*XNUS*KP-4*C22*D53*W3*P2Y1*XNUS*KP-4*C22*D53
X*W2*P2Y2*XNUS*KP-4*C22*W3*D43*P2Y2*XNUS*KP-4*C22*W3*D53Y*P2Y2*
XXNUS*KP
 FTOUT=FTOUT-4*C22*W3*D43Y*P2Y3*XNUS*KP-2*C22*W2*D43*P2Y1*XNUS*KP-2
X*C22*W2*D43*P2Y3*XNUS*KP-4*C22*W2*D53Y*P2Y3*XNUS*KP-2*C22*W2*D43Y
X*P2Y2*XNUS*KP-2*C22*W2*D43Y*P2Y4*XNUS*KP+2*C28*D53*W2*P2Y0*XNUS*
XKP-2*C28*D53*W2*P2Y2*XNUS*KP+2*C28*W2*D53Y*P2Y1*XNUS*KP-2*C28*W2*
XD53Y*P2Y3*XNUS*KP-4*C24*D53*W3*P2Y0*XNUS*KP-4*C24*D53*W2*P2Y1*
XXNUS*KP-4*C24*W3*D43*P2Y1*XNUS*KP-4*C24*W3*D53Y*P2Y1*XNUS*KP-4*
XC24*W3*D43Y*P2Y2*XNUS*KP-2*C24*W2*D43*P2Y0*XNUS*KP-2*C24*W2*D43*
XP2Y2*XNUS*KP-4*C24*W2*D53Y*P2Y2*XNUS*KP-2*C24*W2*D43Y*P2Y1*XNUS*
XKP-2*C24*W2*D43Y*P2Y3*XNUS*KP-2*D53*W3*C29*P2Y0*XNUS*KP+2*D53*W3*
XC29*P2Y2*XNUS*KP
 FTOUT=FTOUT-2*D53*W2*C29*P2Y1*XNUS*KP+2*D53*W2*C29*P2Y3*XNUS*KP+2*
XD53*C27*P1Y0-2*W3*C29*D53Y*P2Y1*XNUS*KP+2*W3*C29*D53Y*P2Y3*XNUS*
XKP-2*W2*C29*D53Y*P2Y2*XNUS*KP+2*W2*C29*D53Y*P2Y4*XNUS*KP
```

Appendix D

Nuclear Matrix Elements in the Closure - Harmonic Oscillator Model

In the standard theory of radiative muon capture the evaluation of :

$$\langle O_1\rangle\langle O_2\rangle^* \equiv \sum_b \langle b|\sum_j \tau_-(j)\exp(-i\vec{s}\cdot\vec{r}_j)\,O_1(j)|a\rangle$$
$$\times \langle b|\sum_k \tau_-(k)\exp(-i\vec{s}\cdot\vec{r}_k)\,O_2(k)|a\rangle^* \qquad \text{D-1}$$

is a principal concern, where $|a\rangle$, $|b\rangle$ are the initial and final nuclear states, respectively, and O(j) is an operator referring to the j'th nucleon. To evaluate products of nuclear matrix elements of the form D-1 the closure relation :

$$\sum_b |b\rangle\langle b| = 1 \qquad \text{D-2}$$

was utilized and further it was assumed the initial state is a spherical, spin zero nucleus which can be described by the shell model as a set of completely filled shells. Accordingly, a Slater determinant was chosen for $|a\rangle$:

$$|a\rangle = \frac{1}{\sqrt{A!}}\sum_P (-)^P \varphi_{\alpha_1}(\vec{r}_1)\,\varphi_{\alpha_2}(\vec{r}_2)\ldots\varphi_{\alpha_A}(\vec{r}_A) \qquad \text{D-3}$$

Here A is the nucleon number and P is the coordinate permutation operator. φ_{α_k} is the wave function for a particle in the shell model state α_k. Hence :

$$\langle O_1\rangle\langle O_2\rangle^* = 2\Big\{\sum_{\lambda_1}\langle\lambda_1|O_2^{\dagger}O_1|\lambda_1\rangle - \sum_{\lambda_1\neq\lambda_2}\langle\lambda_2|\exp(-i\vec{s}\cdot\vec{r})\,O_1|\lambda_1\rangle$$
$$\times\langle\lambda_2|\exp(-i\vec{s}\cdot\vec{r})\,O_2|\lambda_1\rangle^*\Big\} \qquad \text{D-4}$$

where the index λ_1 (λ_2) takes on all values of the coordinate space quantum numbers describing occupied proton (neutron) states.

In our study of RMC the following products of expectation values arose :

1. $<1><1>^*$
2. $\vec{A}\cdot<\vec{P}><1>^*$
3. $<\vec{P}>\cdot<\vec{P}>^*$
4. $\vec{A}\cdot<\vec{P}>\ \vec{B}\cdot<\vec{P}>^*$
5. $<\vec{P}\cdot\vec{P}><1>^*$
6. $\vec{A}\cdot<\vec{P}\ \vec{B}\cdot\vec{P}><1>^*$
7. $\vec{A}\cdot<\vec{P}><\vec{B}\cdot\vec{P}\ \vec{C}\cdot\vec{P}>^*$
8. $<\vec{P}>\cdot<\vec{P}\ \vec{A}\cdot\vec{P}>^*$
9. $<\vec{A}\cdot\vec{P}><\vec{P}\cdot\vec{P}>^*$

where $\vec{P}=-i\vec{\nabla}$ and $\vec{A}$, $\vec{B}$, $\vec{C}$ are vectors not depending on the nucleon coordinates. Expressions for the combinations 1 and 2 can be found in the literature (Lu 63), whereas the quantities 3 through 9 have not appeared in previous RMC studies. Expressions for the latter, which appear here because we keep all $O(1/m^2)$ terms, were derived as follows. Harmonic oscillator functions were taken for the single particle wave functions and the evaluation of expectation values was done in Cartesian coordinates. The unnormalized wave functions are :

$$\varphi_{n_x n_y n_z} \sim exp\left(-(x^2+y^2+z^2)/2\right) H_{n_x}(x) H_{n_y}(y) H_{n_z}(z) \qquad \text{D-5}$$

with H_n a Hermite polynomial of degree n. Each expectation

value required that an integral of the form :

$$I_{mn} = \int_{-\infty}^{\infty} \exp(-isx)\exp(-x^2)\,H_m(x)\,H_n(x)\,dx$$

D-6

be evaluated. The necessary integrals were derived from the relation :

$$\int_{-\infty}^{\infty} \exp(-isx)\exp(-x^2)\,dx = \sqrt{\pi}\,\exp(-s^2/4)$$

D-7

by successively differentiating under the integral sign with respect to s to generate each term in the polynomial $H_m(x)\,H_n(x)$. The final results involved polynomials in sb multiplied by the Gaussian factor $\exp(-(sb)^2/4)$.

The products of expectation values containing an odd number of $\vec{P}$ operators (2,7,8,9) were evaluated by expressing them as combinations of other expectation values given in the list above. A parity argument relates the expectation values. By way of illustration of this relation the evaluation of $\mathrm{Re}\langle\vec{P}\rangle\langle 1\rangle^*$ is considered below. The extension of the argument to the evaluation of the products listed as 7, 8, and 9 is straightforward.

$$\mathrm{Re}\langle\vec{P}\rangle\langle 1\rangle^* = \frac{1}{2}\left\{\langle\vec{P}\rangle^*\langle 1\rangle + \langle\vec{P}\rangle\langle 1\rangle^*\right\}$$

$$= \frac{1}{2}\Big\{-2\sum_{\lambda_1\neq\lambda_2}\langle\lambda_2|\exp(i\vec{s}\cdot\vec{r})|\lambda_1\rangle\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})\vec{P}|\lambda_2\rangle$$

$$-2\sum_{\lambda_1\neq\lambda_2}\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})|\lambda_2\rangle\langle\lambda_2|\vec{P}\exp(i\vec{s}\cdot\vec{r})|\lambda_1\rangle$$

$$= -\sum_{\lambda_1\neq\lambda_2}\langle\lambda_2|\exp(i\vec{s}\cdot\vec{r})|\lambda_1\rangle\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})\vec{P}|\lambda_2\rangle$$

$$+\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})|\lambda_2\rangle\langle\lambda_2|\exp(i\vec{s}\cdot\vec{r})\vec{P}|\lambda_1\rangle$$

$$+\vec{s}\,\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})|\lambda_2\rangle\langle\lambda_2|\exp(i\vec{s}\cdot\vec{r})|\lambda_1\rangle$$

D-8

Note that there is no one-body contribution to $\mathrm{Re}\langle\vec{P}\rangle\langle 1\rangle^*$ since the parent nucleus is assumed to be spherically symmetric. Now change $\vec{r}$ to $-\vec{r}$ in the second term, assuming the states λ_1, λ_2 have definite parity :

$$\mathrm{Re}\,\langle\vec{P}\rangle\langle 1\rangle^* = -\sum_{\lambda_1\neq\lambda_2} \vec{s}\,|\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})|\lambda_2\rangle|^2 + \langle\lambda_2|\exp(i\vec{s}\cdot\vec{r})|\lambda_1\rangle\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})\vec{P}|\lambda_2\rangle - \langle\lambda_1|\exp(i\vec{s}\cdot\vec{r})|\lambda_2\rangle\langle\lambda_2|\exp(-i\vec{s}\cdot\vec{r})\vec{P}|\lambda_1\rangle$$

D-9

If N=Z and states λ_1 are identical with states λ_2, λ_1 and λ_2 can be interchanged, making the last two terms vanish :

$$\mathrm{Re}\,\langle\vec{P}\rangle\langle 1\rangle^* = -\vec{s}\sum_{\lambda_1\neq\lambda_2} |\langle\lambda_1|\exp(-i\vec{s}\cdot\vec{r})|\lambda_2\rangle|^2 = \frac{\vec{S}}{2}\,|\langle 1\rangle|^2_{TB} \qquad \text{D-10}$$

Thus $\mathrm{Re}\langle\vec{P}\rangle\langle 1\rangle^*$ is expressed in terms of the two-body part of $|\langle 1\rangle|^2$.

The computer language REDUCE2 (He 73) was used to sum the polynomials involved in each product of expectation values. Final results for the products of nuclear matrix elements are :

1). $$|\langle 1\rangle|^2 = 20\left\{1-\left(1+\frac{\eta^4}{8}-\frac{\eta^6}{80}+\frac{\eta^8}{640}\right)\exp(-\eta^2/2)\right\} \equiv POL1$$

2). $$\mathrm{Re}\ \vec{A}\cdot\frac{\langle\vec{P}\rangle}{m}\langle 1\rangle^* = -10\,\frac{\vec{A}\cdot\vec{S}}{m}\left(1+\frac{\eta^4}{8}-\frac{\eta^6}{80}+\frac{\eta^8}{640}\right)\exp(-\eta^2/2) \equiv -\frac{\vec{A}\cdot\vec{S}}{2m}\times POL2$$

3). $$\frac{|\langle\vec{P}\rangle|^2}{m^2} = \frac{30}{(mb)^2}\left\{2-\left(1+\frac{\eta^2}{2}-\frac{\eta^4}{120}+\frac{7}{240}\eta^6-\frac{\eta^8}{480}\right)\exp(-\eta^2/2)\right. \equiv POL3$$

4). $$\frac{\langle \vec{A}\cdot\vec{P}\rangle}{m}\,\frac{\langle \vec{B}\cdot\vec{P}\rangle^{*}}{m} = \frac{10}{(mb)^2}\,\vec{A}\cdot\vec{B}^{*}\left\{2-\left(1+\frac{\eta^2}{2}-\frac{\eta^4}{40}+\frac{\eta^6}{80}\right)\exp(-\eta^2/2)\right\}$$

$$-\,\frac{\vec{A}\cdot\vec{S}}{m}\,\frac{\vec{B}^{*}\cdot\vec{S}}{m}\left(\frac{\eta^2}{2}+\frac{\eta^4}{2}-\frac{\eta^6}{16}+\frac{\eta^8}{128}\right)\exp(-\eta^2/2)$$

$$\equiv \vec{A}\cdot\vec{B}^{*}\times POL4-\frac{\vec{A}\cdot\vec{S}}{2m}\,\frac{\vec{B}^{*}\cdot\vec{S}}{2m}\times POL5$$

5). $$\frac{\langle \vec{P}\cdot\vec{P}\rangle}{m^2}\langle 1\rangle^{*} = \frac{60}{(mb)^2}\left\{1-\left(1-\frac{\eta^2}{20}+\frac{31}{240}\eta^4-\frac{\eta^6}{480}-\frac{\eta^8}{3840}\right)\exp(-\eta^2/2)\right\}$$

$$\equiv POL6$$

6). $$\frac{\langle \vec{A}\cdot\vec{P}\,\vec{B}\cdot\vec{P}\rangle}{m^2}\langle 1\rangle^{*} = \frac{20}{(mb)^2}\,\vec{A}\cdot\vec{B}\left\{1-\left(1-\frac{\eta^2}{12}+\frac{7}{48}\eta^4-\frac{\eta^6}{60}+\frac{7}{3840}\eta^8\right)\exp(-\eta^2/2)\right\}$$

$$-\,2\,\frac{\vec{A}\cdot\vec{S}}{m}\,\frac{\vec{B}\cdot\vec{S}}{m}\left(1-\frac{\eta^2}{2}+\frac{7}{16}\eta^4-\frac{\eta^6}{16}+\frac{\eta^8}{256}\right)\exp(-\eta^2/2)$$

$$\equiv \vec{A}\cdot\vec{B}\times POL7-\frac{\vec{A}\cdot\vec{S}}{2m}\,\frac{\vec{B}\cdot\vec{S}}{2m}\times POL8$$

7). $$\mathrm{Re}\,\frac{\langle \vec{A}\cdot\vec{P}\rangle^{*}\langle \vec{B}\cdot\vec{P}\,\vec{C}\cdot\vec{P}\rangle}{m^3} = \frac{\vec{A}\cdot\vec{S}}{2m}\left\{\vec{B}\cdot\vec{C}\times POL7-20\frac{\vec{B}\cdot\vec{C}}{(mb)^2}-\frac{\vec{B}\cdot\vec{S}}{2m}\,\frac{\vec{C}\cdot\vec{S}}{2m}\times POL8\right\}$$

$$+\,\frac{\vec{B}\cdot\vec{S}}{2m}\left\{\vec{A}\cdot\vec{C}\times POL4-20\frac{\vec{A}\cdot\vec{C}}{(mb)^2}-\frac{\vec{A}\cdot\vec{S}}{2m}\,\frac{\vec{C}\cdot\vec{S}}{2m}\times POL5\right\}$$

$$+\,\frac{\vec{C}\cdot\vec{S}}{2m}\left\{\vec{A}\cdot\vec{B}\times POL4-20\frac{\vec{A}\cdot\vec{B}}{(mb)^2}-\frac{\vec{A}\cdot\vec{S}}{2m}\,\frac{\vec{B}\cdot\vec{S}}{2m}\times POL5\right\}$$

$$+\,\frac{\vec{A}\cdot\vec{S}\;\vec{B}\cdot\vec{S}\;\vec{C}\cdot\vec{S}}{8m^3}\times POL2$$

8). $$\mathrm{Re}\,\frac{\langle \vec{P}\rangle^{*}\cdot\langle \vec{P}\,\vec{A}\cdot\vec{P}\rangle}{m^3} = \frac{\vec{A}\cdot\vec{S}}{2m}\left\{POL7-\frac{100}{(mb)^2}-\frac{S^2}{4m^2}\times POL8+POL3\right.$$

$$\left.+\,POL4-\frac{S^2}{4m^2}\times POL5+\frac{S^2}{2m^2}\times POL2\right\}$$

9). $$\mathrm{Re}\,\frac{\langle\vec{A}\cdot\vec{P}\rangle\langle\vec{P}\cdot\vec{P}\rangle^*}{m^3} = \frac{\vec{A}\cdot\vec{S}}{2m}\left\{POL6 + 2\times POL4 - \frac{100}{(mb)^2} + \frac{S^2}{2m^2}(POL2 - POL5)\right\}$$

10). $$POL\emptyset \equiv \frac{20}{(mb)^2}$$

D-11

In our GDR model calculations, the elastic form factor F_{el} and the unretarded dipole part D of the main nuclear matrix element squared were also required. Calculation of these quantities proceeded as above for the matrix elements. The results are :

$$D \equiv \int \frac{d\Omega_\gamma}{4\pi} \langle a| \sum_{i=1}^{A} \sum_{j=1}^{A} J_-(i) J_+(j) (\vec{S}\cdot\vec{r}_i)(\vec{S}\cdot\vec{r}_j) |a\rangle$$

$$= \frac{A}{4}\eta^2 \qquad \text{D-12}$$

$$F_{el} \equiv \frac{1}{A} \langle a| \sum_{i=1}^{A} \exp(i\vec{S}\cdot\vec{r}_i) |a\rangle$$

$$= (1 - \frac{1}{4}\eta^2 + \frac{1}{80}\eta^4) \exp(-\eta^2/4) \qquad \text{D-13}$$

Appendix E

Main Nuclear Matrix Element Squared Using the SBL Wave Function

Taking the ground state wave function to be a Slater determinant :

$$|a\rangle = \frac{1}{\sqrt{A!}} \sum_P (-)^P \varphi_{\alpha_1}(\vec{r}_1)\, \varphi_{\alpha_2}(\vec{r}_2) \ldots \varphi_{\alpha_A}(\vec{r}_A) \qquad \text{E-1}$$

and substituting the SBL (Sh 73) single-particle wave functions :

$$\varphi_\alpha(\vec{r}) \equiv \varphi_{Nljm_jt}(\vec{r}) = \sum_M \sum_{m_l, m_s} C_{NMlj} \langle l m_l \tfrac{1}{2} m_s | j m_j \rangle R^t_{Ml}(r) \times Y_l^{m_l}(\Omega_r)\, \alpha_s \alpha_t$$

$$= \sum_M \sum_{m_l, m_s} C_{NMlj} \langle l m_l \tfrac{1}{2} m_s | j m_j \rangle |M l m_l\rangle\, |\tfrac{1}{2} m_s\rangle |\tfrac{1}{2} m_t\rangle \qquad \text{E-2}$$

in Eq. D-1 yields, in closure approximation :

$$\langle 1\rangle\langle 1\rangle^* = \sum_{\lambda_1} \langle \lambda_1 | \mathbb{1}\mathbb{1}^\dagger | \lambda_1 \rangle - \sum_{\lambda_1 \neq \lambda_2} \langle \lambda_2\, e^{-i\vec{s}\cdot\vec{r}} | \lambda_1 \rangle \langle \lambda_2 | e^{-i\vec{s}\cdot\vec{r}} | \lambda_1 \rangle^* \qquad \text{E-3}$$

where λ_1 (λ_2) takes on all values of the quantum numbers ($Nljm_j$) of states occupied by protons (neutrons). The C_{NMlj} are constant coefficients determined by Shao et al (Sh 73) and reproduced in table VI, $\langle l m_l \tfrac{1}{2} m_s | j m_j \rangle$ are Clebsch-Gordon coefficients, and $|M l m_l\rangle$, $|\tfrac{1}{2} m_s\rangle$, $|\tfrac{1}{2} m_t\rangle$ are space, spin, isospin wave functions, respectively. Working in spherical coordinates, the angular momentum algebra can be handled using standard techniques (Ed 57), so that a straightforward calculation yields :

Table VI

Oscillator Function Coefficients C_{NMlj} for ^{40}Ca

(Normalization=1)

M \ Nlj	1S 1/2	2S 1/2	1P 3/2	1P 1/2	1D 5/2	1D 3/2
1	0.9575	0.1709	0.9867	0.9994	0.9947	0.9951
2	-0.2634	0.8223	-0.1247	-0.0030	-0.0730	-0.0692
3	-0.1141	-0.5005	-0.0958	-0.0289	-0.0527	0.0684
4	-0.0239	0.1592	-0.0419	-0.0163	-0.0500	-0.0160
5	0.0059	-0.1262	0.0000	-0.0066	-0.0083	0.0119
6	0.0086	0.0536				

Table VI. Oscillator function coefficients C_{NMlj} for the ^{40}Ca wave function of Shao et al (Sh 73). The single particle wave functions are normalized to one.

$$\langle 1\rangle\langle 1\rangle^* = Z - \sum_{\substack{M_1,M_2\\M_1',M_2'}} \sum_{\substack{N_1 l_1 j_1\\N_2 l_2 j_2}} \sum_{l} C_{N_1M_1l_1j_1} C_{N_2M_2l_2j_2} C_{N_1M_1'l_1j_1} C_{N_2M_2'l_2j_2} (2l+1)(2j_1+1) \times (2j_2+1) I_{M_1l_1M_2l_2l} I^*_{M_1'l_1M_2'l_2l} \begin{pmatrix} j_1 & j_2 & l \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \quad \text{E-4}$$

The last factor is the square of a Wigner 3-j symbol and the radial integrals are represented as :

$$I_{MlM'l'L} = \int R^-_{Ml}(r) R^+_{M'l'}(r) j_L(sr) r^2 dr \quad \text{E-5}$$

with $j_L(sr)$ a spherical Bessel function. For the harmonic oscillator potential, Eq. 4.97 of DeForest et al (De 66) gives :

$$I_{MlM'l'L} = \frac{2^L}{(2L+1)!!} \left(\frac{\eta^2}{2}\right)^{\frac{L}{2}} \exp\left(-\frac{\eta^2}{2}\right) \left((M-1)!(M'-1)!\right)^{\frac{1}{2}} \left(\Gamma(M+l+\tfrac{1}{2})\Gamma(M'+l'+\tfrac{1}{2})\right)^{\frac{1}{2}}$$
$$\times \sum_{N=0}^{M-1} \sum_{N'=0}^{M'-1} \frac{(-)^{N+N'}}{N!N'!} \frac{1}{(M-N-1)!(M'-N'-1)!} \frac{\Gamma(\frac{1}{2}(l+l'+2N+2N'+L+3))}{\Gamma(N+l+3/2)\Gamma(N'+l'+3/2)}$$
$$\times F\left(\tfrac{1}{2}(L-l-l'-2N-2N'); L+3/2; \eta^2/2\right) \quad \text{E-6}$$

where $\eta = (sb)$, Γ is the gamma function, and F is the confluent hypergeometric function.

Taking $C_{NMlj} = \delta_{NM}$ recovered the simple wave functions used in Appendix D and allowed us to check that for this simpler case, Eq. E-4 reduced to the appropriate expression which appears as Eq. 4.10 in Luyten et al (Lu 63).

Inspection of Eq. E-6 shows that the evaluation of Eq. E-4 must lead to a result of the form :

$$\langle 1\rangle\langle 1\rangle^* = Z - Z(a_0 + a_1\eta^2 + a_2\eta^4 + \ldots)\exp(-\eta^2/2) \quad \text{E-7}$$

hence our task was to calculate the a_i. This was accomplished by a FORTRAN program which was checked insofar as possible by letting $C_{NMlj} = \delta_{NM}$, thus recovering the

corresponding expression obtained for the simple wave functions. Our final result was :

$$\langle 1\rangle\langle 1\rangle^{*} = 20-(20-3.05\eta^{2}+5.15\eta^{4}-1.63\eta^{6}+0.414\eta^{8})\,e^{-\eta^{2}/2} \qquad \text{E-8}$$

For the GDR model, it was necessary to also calculate the unretarded dipole part D of the main nuclear matrix element and the elastic form factor F_{el}. These calculations proceeded in a wholly analogous fashion to the one above and hence only the final results appear here.

$$D = 13.05\,\eta^{2} \qquad \text{E-9}$$

$$F_{el} = (1-0.346\,\eta^{2}+0.0386\,\eta^{4}-0.00416\,\eta^{6})\,e^{-\eta^{2}/4} \qquad \text{E-10}$$

Appendix F

A Comment on Fearing's Theorem

The theorem proved by Fearing (Fe 75) states that :

"for the standard theory of radiative muon capture including all weak couplings except g_s and all usual diagrams, the photon asymmetry α and the circular polarization β satisfy $\alpha=+1+O(1/m^2)$ and $\beta=+1+O(1/m^2)$, where m is the nucleon mass."

The standard theory referred to (Ro 65, Fe 66) considers the muon and nucleon to propagate freely in their intermediate states. In what follows the question of whether the theorem remains valid when external potentials are incorporated in the theory is examined.

Consider the Feynman diagram in figure 1(a). The corresponding amplitude can be written :

$$M_1^F = \bar{u}_\nu \gamma_\alpha (1-\gamma_5) i S_0 (-i\not{\epsilon}) u_\mu T^\alpha \tag{F-1}$$

where S_0 is the free muon propagator and T^α the hadronic current. This amplitude forms part of the standard theory considered by Fearing (Fe 75). If figure 1(a) is altered so that the intermediate state muon interacts with some external field, then to first order in the coupling to the external field the new amplitude is :

$$M_1 = \bar{u}_\nu \gamma_\rho (1-\gamma_5) i S_0 (q+\mu-k) (-i\Gamma(q)) i S_0 (\mu-k) (-i\not{\epsilon}) u_\mu T^\rho \tag{F-2}$$

with $\Gamma(q)$ the external interaction vertex. Note that to all orders of a Lippmann-Schwinger expansion of the muon

propagator, the rightmost factor will be $S_0(\mu-k)$. (This fact will be used later when our argument is generalized.) All other diagrams of interest (figs. 1(b)-1(e), modified to include the external field) may be lumped into the amplitude :

$$M_2 = \bar{u}_\nu \gamma_\alpha (1-\gamma_5) u_\mu R^{\alpha\beta}_{\ell} \epsilon_\beta \qquad \text{F-3}$$

where $R^{\alpha\beta}_{\ell}\epsilon_\beta$ represents photon emission by the hadronic current. Using Eqs. F-2 and F-3 yields :

$$M^2 \equiv \sum_{spins} |M_1 + M_2|^2$$

$$= Tr\, \gamma_\rho (1-\gamma_5)([S_0(q+\mu-k)T(q)S_0(\mu-k)]\not{\epsilon}\, T^\ell + R^{\ell\beta}\epsilon_\beta \frac{(\not{\mu}+m_\mu)}{2m_\mu}\frac{(1+\gamma_5\not{s})}{2}$$

$$\times(\not{\epsilon}^*[S_0(\mu-k)T^*(q)S_0(q+\mu-k)]T^{*\delta} + R^{*\delta\eta}\epsilon^*_\eta)\gamma_\delta(1-\gamma_5)\frac{\not{\nu}}{2m_\nu} \qquad \text{F-4}$$

Taking the muon at rest and using the explicit form for the free muon propagator :

$$S_0(\mu-k) = \frac{(\not{\mu}-\not{k}+m_\mu)}{(\mu-k)^2 - m_\mu^2} = \frac{\not{\mu}-\not{k}+m_\mu}{-2k\, m_\mu} \qquad \text{F-5}$$

yields, after some algebra :

$$M^2 = Tr\, \not{\nu}\gamma_\rho(1-\gamma_5)([S_0(q+\mu-k)T(q)]\frac{(\lambda-\gamma_5)}{2m_\mu}\frac{(1+\gamma_0)}{2}\vec{\sigma}_L\cdot\hat{\epsilon}\, T^\ell + R^{\ell\beta}\epsilon_\beta)$$

$$\times(1+\vec{\sigma}_L\cdot\vec{s})(\vec{\sigma}_L\cdot\hat{\epsilon}^*\frac{(\lambda+\gamma_5)}{2m_\mu}[T^*(q)S_0(q+\mu-k)T^{*\delta} + R^{*\delta\eta}\epsilon^*_\eta)\gamma_\delta\frac{(1-\gamma_5)}{4m_\nu} \qquad \text{F-6}$$

Now consider the factor $(S_0(q+\mu-k)T(q))$. If higher order terms from the expansion of S were kept in M_1, this factor would be different. Independent of its explicit form however, it can always be decomposed into a sum E+O such

that :

$$\gamma_5 E = E \gamma_5$$

$$\gamma_5 O = - O \gamma_5$$

F-7

Representing the factor in question by the sum E+O and using the commutation rules of Eq. F-7 allows us to write :

$$M^2 = \mathrm{Tr}\, \not{p}\, \gamma_\rho (1-\gamma_5)\left(\left[E \frac{\lambda_+}{m_\mu} - O \frac{\lambda_-}{m_\mu}\right] \frac{(1+\gamma_0)}{2}\, \vec{\sigma}_L \cdot \hat{\epsilon}\, T^\rho + R^{\rho\rho}\epsilon_\rho\right)(1+\vec{\sigma}_L \cdot \vec{\xi})$$

$$\times \left(\vec{\sigma}_L \cdot \hat{\epsilon}^* \left[\frac{\lambda_+}{m_\mu} E^* - \frac{\lambda_-}{m_\mu} O^*\right] T^{*\delta} + R^{*\delta\eta}\epsilon^*_\eta\right) \frac{\gamma_\delta}{2m_\nu} \qquad \text{F-8}$$

Now look at the first term, which is the square of the muon radiating amplitude. Taking explicit representations for the gamma matrices and using $\hat{\epsilon} = (\hat{i} - i\lambda\hat{j})/\sqrt{2}$, the term becomes :

$$M_{\mu r}^2 = \mathrm{Tr}\, \not{p}\, \gamma_\rho (1-\gamma_5) E \frac{(1+\gamma_0)}{2} T^\rho \lambda_+ (1-\vec{\sigma}_L \cdot \hat{k})(1+\vec{\xi}\cdot\hat{k}) E^* T^{*\delta} \frac{\gamma_\delta}{2m_\nu m_\mu^2}$$

$$+ \mathrm{Tr}\, \not{p}\, \gamma_\rho (1-\gamma_5) O \frac{(1+\gamma_0)}{2} T^\rho \lambda_- (1+\vec{\sigma}_L \cdot \hat{k})(1-\vec{\xi}\cdot\hat{k}) O^* T^{*\delta} \frac{\gamma_\delta}{2m_\nu m_\mu^2} \qquad \text{F-9}$$

Contact is made with the work of Fearing (Fe 75), in which the muon was taken to propagate freely, by setting $E=E^*=1$ and $O=O^*=0$. In this case M, has the overall factor $\lambda_+(1+\vec{\xi}\cdot\hat{k})$ and thus has $\alpha=1$. In the general case where the muon propagates in some external field however, O will not vanish. For this general case then, the muon radiating diagram will contribute to $\alpha\neq1$. Thus the conclusion is reached that Fearing's theorem does not apply to theories of radiative muon capture which include propagation of the muon in external potentials.